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ENGINEERING APPLICATIONS OF ANALOG COMPUTERS

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Louis C. Just and Alan L. Winiecki

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ANL-6319 Rev.
Mathematics and Computers
(TID-4500, 18th Ed.)
AEC Research and
Development Report

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois

ENGINEERING APPLICATIONS OF ANALOG COMPUTERS

by

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Revised October 1962

(Originally published February 1961)

Operated by The University of Chicago
under
Contract W-31-109-eng-38
with the
U. S. Atomic Energy Commission

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ENGINEERING APPLICATIONS OF ANALOG COMPUTERS

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INTRODUCTION

This publication is an extension of Bryant, L. T., Just, L. C., and Pawlicki, G. S., Introduction to Electronic Analog Computing, ANL-6187 (July 1960). Six experiments are presented from the fields of reactor engineering, heat transfer, and dynamics.

The mathematical representation for most of these experiments is in the form of nonlinear differential equations. In usual practice simplifying assumptions are introduced to linearize the equations. This linearization may alter the mathematical model sufficiently to cast doubt upon its applicability. If an analog computer is available, the nonlinear equation may be solved directly.

The presentation of these experiments has been designed to provide insight into physical phenomena and their mathematical representation. The steps required for producing the analog solution will be shown, as well as complete information for duplicating the solution. Graphical results are provided.

The format of each experiment will be:

1. Description of the problem
2. Mathematical statement of the problem including:
    a. Constants
    b. Initial Conditions
3. Preparation of machine equations
    a. Machine Variables
    b. Scale Factors
4. Analog circuit diagram
    a. Flow Sheet
    b. Potentiometer setting sheet
    c. Static Check sheet
5. Graphical representation of the solution.
6. Bibliography

I. DECELERATION OF A REACTOR CONTROL ROD

1. Problem Description

When a control rod is suddenly inserted or rejected from the core of a reactor, the rapid motion is quickly dampened by a dashpot or buffer mechanism, usually consisting of a hydraulic system which prevents sudden shock of the control drive mechanisms.

Constant deceleration-type dashpots give the most favorable characteristics for protection against shock loads. Essentially, a piston moves through oil, and the oil is squeezed into small clearances; this process in turn develops large amounts of frictional resistance. This friction, which is proportional to the speed of the moving piston, instigates the retarding force which slowly stops the motion of the control drive.

This hydraulic drag and the ensuing kinetic energy dissipation are frequently described by differential equations. Elias' equation of buffer motion[I-1]* is given by

$$\frac{dV_p}{dX} = - \frac{2 \mu \pi D_p^2 L_d^2 X}{W(L_d C - CX)^2} \quad .$$

Various plots of the buffer characteristics are shown on Figs. 2, 3, 4, and 5.

Many parameters may be investigated before the design conditions for a particular problem are satisfied.[I-2,I-3]

2. Mathematical Statement of the Problem

a. Equations:

$$\frac{dV_p}{dX} = - \frac{2 \mu \pi D_p^2 L_d^2 X}{W(L_d C - CX)^2} \quad . \tag{1}$$

b. Constants and Variables

Symbol	Description	Value	Units
V_p	velocity into the dashpot	Variable	ft/sec
D_p	diameter of the dashpot	2	inches
C	dashpot clearance	0.03	inch
L_d	dashpot length	6	inches
μ	viscosity of the dashpot fluid	Variable	lb/(ft)(sec)
W	weight of the control rod	290	lb
X	distance into the dashpot	6	inches

*References in each section are given at the end of each section.

c. Initial Conditions

At t = 0:

$$X = 0$$

$$\frac{dX}{dt} = 70 \text{ ft/sec}$$

$$\frac{dV_p}{dt} = 0 \quad .$$

d. Analysis of Equations

Since

$$\frac{dV_p}{dX} = \frac{dV_p/dt}{dX/dt} = \frac{d^2X/dt^2}{dX/dt} \quad , \tag{2}$$

the original equation may be restated as

$$\frac{d^2X}{dt^2} = -\left(\frac{2\,\mu\pi D_p^2 L_d^2}{WC^2}\right)\left(X\,\frac{dX}{dt}\right)\left(\frac{1}{L_d - X}\right)^2 \quad . \tag{3}$$

3. Preparation of Machine Equations

a. Machine Variables and Scale Factors

$$X' = bX \quad ; \qquad a = 10^3$$
$$t' = at \quad ; \qquad b = 10^2 \tag{4}$$

b. Scaled Equations

$$\frac{d^2X'}{dt'^2} = -\frac{b}{a^2}\left(\frac{2\,\mu\pi D_p^2 L_d^2}{WC^2}\right)\left(\frac{X'}{b}\right)\left(\frac{dX'}{dt'}\,\frac{a}{b}\right)\left(\frac{b}{L_d b - X'}\right)^2$$

$$= -\frac{b}{a}\left(\frac{2\,\mu\pi D_p^2 L_d^2}{WC^2}\right)\left(X'\,\frac{dX'}{dt'}\right)\left(\frac{1}{L_d b - X'}\right)^2 \quad . \tag{5}$$

c. Machine Equation

When the values of constants and scale factors are introduced into Eq. (5), the machine equation results:

$$\frac{d^2X'}{dt'^2} = -\,(5.07\,\mu)\quad X'\left(\frac{dX'}{dt'}\right)\left(\frac{1}{50 - X'}\right)^2 \quad . \tag{6}$$

The initial conditions (in terms of voltages) are:

$$X' = bX(0) = 0$$

$$\frac{dX'}{dt'} = \frac{b}{a}\frac{dX}{dt} = 7.0 \text{ volts} \quad .$$

$$\frac{d^2X'}{dt'^2} = \frac{b}{a^2}\frac{d^2X}{dt^2} = 0 \quad .$$

4. Analog Circuit Diagram

a. Flow Sheet

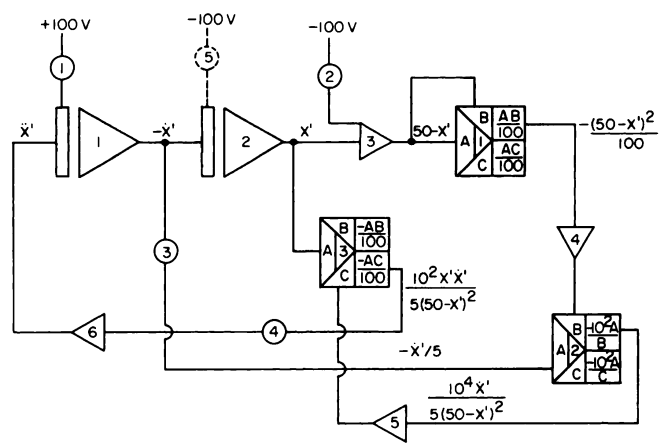

Fig. 1. Circuit Diagram for Solution of Elias' Equation of Buffer Motion

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b. **POTENTIOMETER SETTINGS**

PROBLEM NO. ____
DRAWING NO. ____
DATE ____

REACTOR CONTROL ROD DECELERATION

POTENTIOMETER NO. DRAWING	MACHINE	MATHEMATICAL VALUE	VALUE	CORRECTION	SETTING	SET	PARAMETERS
1		V_p' (volts)	7.00		0700		
2		X' (volts)	-50.00		5000		
3		0.2	0.2000		2000		
4		$\frac{5}{100}(5.07\,\mu)$					For Figs. 2 & 5
		$\mu_1 = 0.0494$	0.0125		0125		
		$\mu_2 = 0.0795$	0.0202		0202		
		$\mu_3 = 0.102$	0.0258		0258		
1		$V_p'(0)$ (volts)					
		$V_1 = 70$ ft/sec	7.00		0700		
		$V_2 = 50$ ft/sec	5.00		0500		
		$V_3 = 20$ ft/sec	2.00		0200		For Figs. 3 & 4
2		X' (volts)	-50.00		5000		
3		0.2	0.2000		2000		
4		$\frac{5}{100}(5.07\,\mu_2)$	0.0202		0202		

Parameters:

$a = 10^3$

$b = 10^2$

$V_p(0) = 70$ ft/sec

$D_p = 2$ in.

$C = 0.03$ in.

$L_d = 6$ in.

$W = 290$ lb

$X = 6$ in.

AMD-2C (8-57)

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c. **STATIC CHECK**

PROBLEM NO.
DRAWING NO.
DATE

REACTOR CONTROL ROD
DECELERATION

UNIT	UNIT NUMBER		OUTPUT (VOLTS)	REMARKS	INTE-GRATOR	INITIAL CONDI-TION	SET	PARAMETERS
	DRAWING	MACHINE						
	1			μ_2 = 0.0795				
POT		1	+ 7.00					
		2	-50.00					
		5	-10.00	FOR STATIC CHECK				
		3	- 1.40					
		4	+ 0.02					
AMP		1	- 7.00					
		2	+10.00					
		3	+40.00					
		4	+16.00					
		5	- 8.75					
		6	- 0.02					
MULT		1	-16.00					
		2	+ 8.75					
		3	+ 0.88					

AMD-2A (8-57)

5. Graphical Results

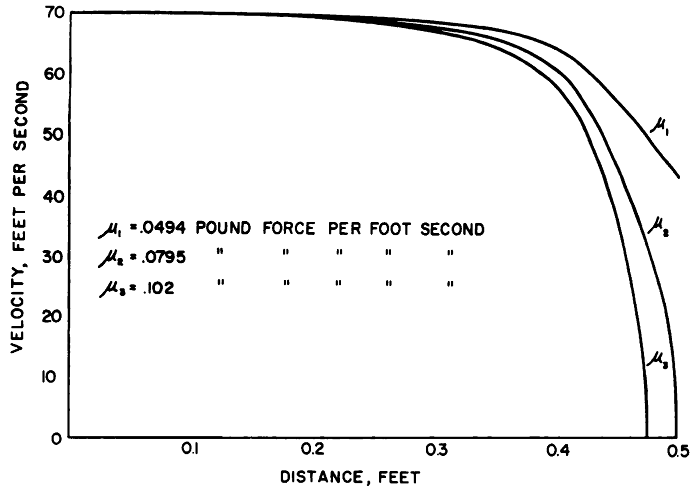
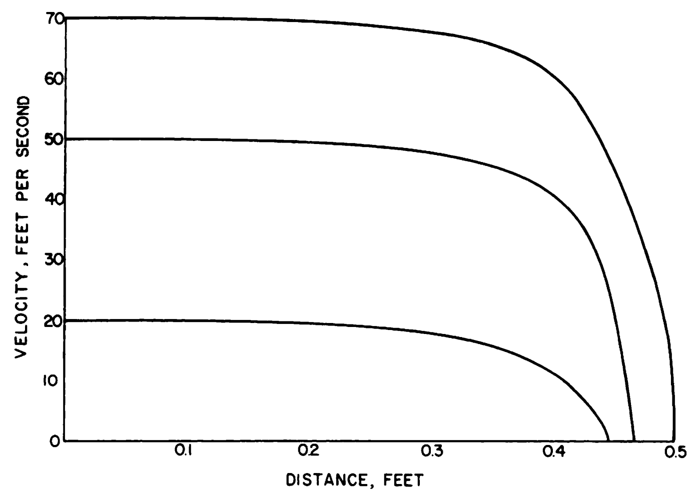


Fig. 2

Velocity Versus Distance for Various Viscosities

Fig. 3

Velocity Versus Distance for Various Initial Velocities. Viscosity is 0.0795 Pound Force per Foot-Second



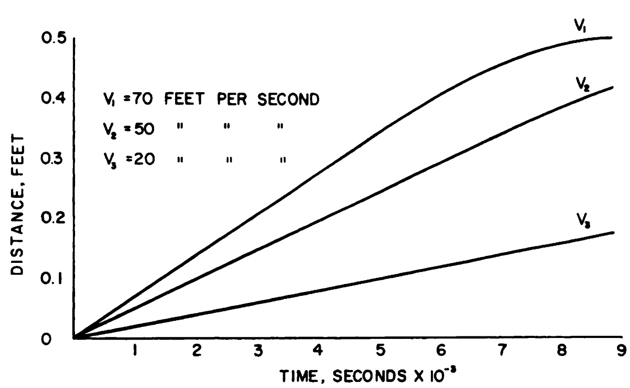


Fig. 4

Distance Versus Time for Various Initial Velocities

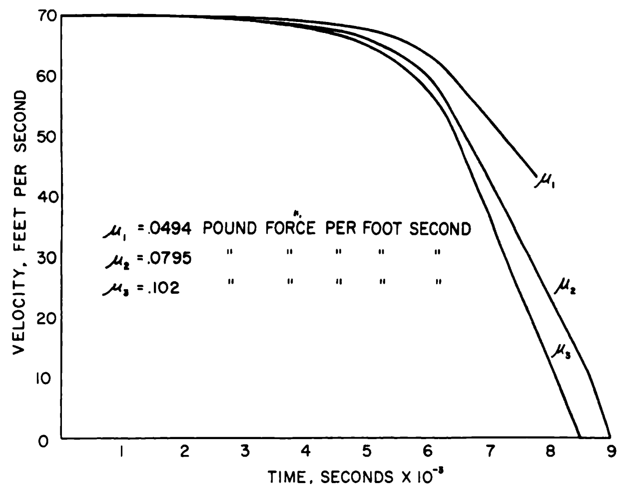


Fig. 5

Velocity Versus Time for Various Viscosities

6. Bibliography

I-1. Freund, G. A. et al., Design Summary Report on the Transient Reactor Test Facility (TREAT), ANL-6034 (June 1960).

I-2. Koch, L. J. et al., Hazard Summary Report Experimental Breeder Reactor II (EBR-II), ANL-5719 (May 1957) p. 30.

I-3. Bishop, A. A., and Berringer, R. T., Hydraulic Shock Absorbers for the Yankee Reactor Control Rods, YAEC-111 (March 1959).

II. PRESSURE VARIATIONS THROUGH A PACKED BED

1. Problem Description

The future applications of nuclear power sources will depend on whether reactor technology can match the demand for higher power densities and higher operating temperatures. A popular concept for advanced application is the packed bed reactor.(II-1) (II-2) Equations of fluid flow and heat transfer for this concept are dependent upon the particular packed-bed system, particle shape, and the fluid for which they are developed.

The solutions to problems for this type of reactor design are usually obtained through use of empirical corrections.(II-3) An equation derived by MacFarlane(II-4) from the basic Bernoulli equation illustrates a fundamental method for calculating the performance of packed bed arrangements. This relationship

$$\frac{dP}{dx} = \frac{(K + Hx)P}{E + Dx - P^2}$$

expresses in differential form the variation of pressure and distance of a packed bed one square foot in cross section. MacFarlane also indicates four other general methods used for calculating laminar fluid flow in packed beds and describes their derivation.

2. Mathematical Statement of the Problem

a. Equations and Constants

$$\frac{dP}{dx} = \frac{(K + Hx)P}{E + Dx - P^2} \qquad (1)$$

$$D = \frac{QP_0G}{g_c \rho_0 T_0 C_p} = 1.73 \times 10^4 \ \frac{lb^2}{ft^5}$$

$$G = \frac{fG^2P_0}{2g_c D_p \rho_0} = 3.439 \times 10^8 \ \frac{lb^2}{ft^5}$$

$$H = \frac{fGQP_0}{2g_c D_p \rho_0 T_0 C_p} = 5.22 \times 10^9 \ \frac{lb^2}{ft^6}$$

$$E = G^2P_0 / g_c \rho_0 = 1130 \ \frac{lb^2}{ft^4}$$

$$K = D + G = 3.44 \times 10^8 \ \frac{lb^2}{ft^5}$$

Q	=	Volumetric heat generation rate	=	$5\ Mw/ft^3$
G	=	Mass flow rate of helium coolant	=	$0.378\ lb/(sec)(ft^2)$
g_c	=	Gravitational constant	=	$32.17\ ft/sec^2$
ρ_0	=	Initial density	=	$0.083\ lb/ft^3$
T_0	=	Initial temperature	=	200°F
P_0	=	Initial pressure	=	10 atmospheres
C_p	=	Specific heat at constant pressure	=	1.25 BTU/(lb)(°F)
D_p	=	Diameter of the particle	=	200 microns
f	=	friction factor, $\alpha + \beta\ (x/L)$	=	$285 + 230\ (x/L)$

b. Initial Conditions

$P_0 = 2.12 \times 10^4\ lb/ft^2$

$x_0 = 0$

$L = 0.2$ ft.

3. Preparation of the Machine Equations

a. Machine Variables and Scale Factors

$$x = t \qquad x_{(final)} = t_{(final)}$$
$$t' = at \qquad dt' = adt \qquad a = 10^2$$
$$P' = bP \qquad dP' = bdP \qquad b = 10^{-3} \qquad (2)$$
$$P'_{(0)} = bP_0 = (10^{-3})(2.12 \times 10^4) = 21.2 \text{ volts},$$
$$t'(final) = at(final) = 10^2(0.2) = 20.0 \text{ volts}$$

b. The Scaled Equation

Substituting equations (2) into equation (1), the scaled equation is obtained:

$$\frac{dP'}{dt'} = \frac{1}{a}\left\{\frac{[K + (H/a)\,t']\,P'}{E + (Dt'/a) - (P'^2/b^2)}\right\} \qquad . \qquad (3)$$

c. The Machine Equation

When numerical values are placed into equation (3), the machine equation (4) results:

$$\frac{dP'}{dt'} = \frac{(3.44 + 0.522\ t')P'}{0.0002\ t' - P'^2} \tag{4}$$

4. Analog Circuit Diagram

a. Flow Sheet

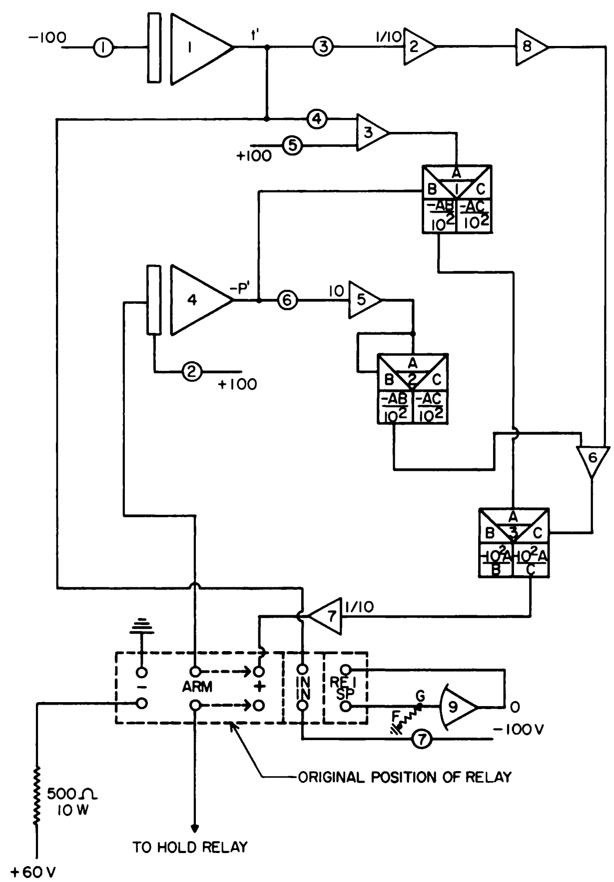


Fig. 6. Circuit Diagram for the Solution of MacFarlane's Equation

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b. **POTENTIOMETER SETTINGS**

PROBLEM NO. ____
DRAWING NO. ____
DATE ____

PRESSURE REDUCTION THROUGH A PACKED BED

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORRECTION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
1		0.01a volt	-1.00		0100		
2		P_0b volt	+21.2		2120		
3		b^2D/a	0.0002		0002		
4		b^2H/a^2	0.5220		5220		
5		b^2K/a	3.44		0344		
6		$\sqrt{10}$	3.162		3162	(10)	
7		aL	-20.00		2000		

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c. **STATIC CHECK**

PROBLEM NO. ____
DRAWING NO. ____
DATE ____

PRESSURE REDUCTION THROUGH A PACKED BED

UNIT	UNIT NUMBER		OUTPUT (VOLTS)	REMARKS	INTEGRATOR	INITIAL CONDITION	SET	PARAMETERS
	DRAWING	MACHINE						
AMP	1		+1.00					
	2		0.0					
	3		-3.96					
	4		-21.2					
	5		+66.9					
	6		44.7					
	7		-0.188					
	8		0.0					
POT	3		0.0					
	4		0.52					
	6		-6.69					
MULT	1		-0.84					
	2		-44.7					
	3		+1.88					

5. Graphical Results

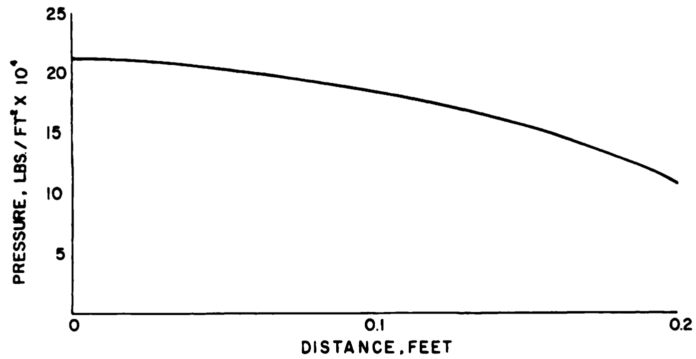


Fig. 7. Pressure Versus Distance

6. Bibliography

II-1. Leroy, M. M., and Newgard, J. J., Pebble Bed Nuclear Reactor for Space Vehicle Propulsion, Aero/Space Engineering, 19 (4), 54-58 (April 1960).

II-2. Robinson, S. T., and Bevenati, R. F., A High Temperature Gas Cycle Pebble Bed for Central Station Use, TID-7564 (1958).

II-3. Lancashire, R. B., Leyberg, E. A., and Morris, J. F., Heat-transfer Coefficients for a Full-scale Pebble-Bed Heater, Ind. Eng. Chem. 52, 433 (1960).

II-4. Rodin, M. B., Study of the Packed-Bed Fuel Element Concept, ANL-6193 (August 1960).

III. REACTOR KINETICS OVER MANY DECADES WITH THERMAL FEEDBACK (SIMULATION OF A TREAT TRANSIENT)

1. Problem Description

The TREAT reactor was designed to generate a very large, transient, thermal flux field of short duration.(III-1) The maximum integrated flux is greater than 10^{15} neutrons/cm^2.

The core is a dispersion of highly enriched uranium (as the oxide or carbide) in a graphite matrix. The graphite serves as a moderator, a heat sink, and a generator of a sizeable negative temperature coefficient. The latter effect is due to the fact that the energy of the thermal neutrons increases with graphite temperature thus causing an increase in the leakage probability.

The purpose of this experiment is to simulate a TREAT transient initiated by control rod withdrawal and terminated by the negative temperature coefficient. Since a large excursion is expected, the reactor kinetics equations will be transformed by a substitution.(III-2)

$$\eta = \ell n \quad n(t)/n(0) \quad .$$

The equations describing the neutron kinetics (with 6 delayed groups) will be solved on the analog computer. They will be forced by changes in K_{ex}.

2. Mathematical Statement of the Problem

a. Equations:

$$\frac{d\eta}{dt} = \frac{\beta}{\ell}\left[(1 - \beta)K_{1ex} + \sum_{i=1}^{6} \frac{\beta_i}{\beta}\psi_i\right]$$

$$\frac{d\psi_i}{dt} = \lambda_i \beta K_{1ex} - \frac{d\eta}{dt} - \psi_i\left(\lambda_i + \frac{d\eta}{dt}\right) \qquad i = 1, \ldots, 6,$$

where

$$\eta = \ell n \; n(t)/n(0)$$

$$\epsilon_i = e^{-\eta}\, C_i(t)/C_i(0)$$

$$\psi_i = \epsilon_i - 1$$

$$K_{1ex} = K_{ex}/\beta$$

b. Constants

$\beta = 0.00755$

$\ell = 8.6 \times 10^{-4}$

i	λ_i	β_i
1	0.01246	0.00025
2	0.0315	0.00166
3	0.1535	0.00213
4	0.456	0.00241
5	1.612	0.00085
6	14.3	0.00025

c. Initial Conditions

$\eta(0) = 0$

$\psi_i = 0$

$K_{1ex} = 0$

3. Preparation of Machine Equations

a. Machine Variables

$t' = at$

$\eta' = b\eta$

$K'_{1ex} = c\,K_{1ex}$

$\psi'_i = d_i \psi_i$

b. Scale Factors

$a = 10$

$b = 2$

$c = 25$

$d_1 = d_2 = \ldots = d_6 = 2$

c. Scaled Equations

$$\frac{d\eta'}{dt'} = \frac{\beta}{a\ell}\left[\frac{b(1-\beta)}{c} K'_{1ex} + \sum_{i=1}^{6} \frac{b\beta_i}{d_i\beta}\psi'_i\right]$$

$$\frac{d\psi'_i}{dt'} = \frac{d_i\lambda_i\beta}{ac} K'_{1ex} - \frac{di}{b}\frac{d\eta'}{dt'} - \psi'_i\left(\frac{\lambda_i}{a} + \frac{1}{b}\frac{d\eta'}{dt'}\right) \qquad i = 1, \ldots, 6.$$

d. The Generation of K'_{1ex}

The expression for K_{ex} is made up of two parts: the contribution of the control rod and the contribution of the negative temperature coefficient, that is,

$$K_{ex} = K(t) - K(n,t) \quad ,$$

where

$$K(t) = \begin{cases} 0.04t \text{ for } 0 \leq t \leq 0.5 \text{ sec} \\ 0.02 \text{ for } t > 0.5 \text{ sec} \end{cases}$$

and

$$K(n,t) = 10^{-10} \int n\, dt \quad .$$

Since

$$K'_{1ex} = \frac{c}{\beta} K_{ex}$$

then

$$K'_{1ex} = \frac{c}{\beta} K(t) - \frac{c}{\beta} K(n,t) \quad ,$$

or, more simply,

$$K'_{1ex} = K'_1(t) - K'_1(n,t) \quad .$$

$$K'_1(t') = \begin{cases} 0.04 \frac{c}{\beta} \frac{t'}{a} \text{ for } 0 \leq t' \leq 0.5a \text{ sec} \\ 0.02 \frac{c}{\beta} \text{ for } t' > 0.5a \text{ sec} \end{cases}$$

$K'_1(t')$ can be generated by means of an integrator and a relay.

The generation of $K_1'(n,t)$ is more complicated: a function generator is needed. If the machine variables and scale factors are substituted into

$$K(n,t) = 10^{-10} \int n dt \quad ,$$

the result is

$$K_1'(n,t) = \frac{c}{\beta a} 10^{-10} \int n dt' \quad .$$

But

$$n = n_1 \, n(0) \quad ;$$

therefore

$$K_1'(n,t) = \frac{c n(0)}{\beta a} 10^{-10} \int n_1 dt \quad .$$

The analog computer will supply $\eta = \ell n \, n_1$ (due to a change in variable) and since $e^{\ell n \, n_1} = n_1$,

$$K_1'(n,t) = \frac{c n(0)}{\beta a} 10^{-10} \int e^{\eta} dt' \quad .$$

After the terms are collected,

$$K_1'(n,t) = c \int e^{\alpha} dt' \quad ,$$

where

$$\alpha = \eta + \ell n \, n(0) - \ell n \, 10^{10} - \ell n \, a\beta \quad .$$

For the values given the constants, and for $n(0) = 10^2$, $\alpha = \eta - 15.836$.

Then $25 \, e^{\alpha}$ will be generated with a diode-function generator (DFG).(III-3) In order to decrease the slope of the function, the DFG will be driven by $10\eta - 100$.

In the actual experiment, the input to integrator 3 is removed (by means of a relay) until $25\, e^{\alpha} = 0.01$ volt.

DFG DATA

η	$10\eta - 100$	α	$25\, e^{\alpha}$
0.0	-100	-15.836	-
8.012	- 19.88	- 7.821	00.01
9.938	- 0.62	- 6.438	00.04
10.42	+ 4.2	- 5.416	00.11
12.0	+ 20.0	- 3.836	00.54
13.0	+ 30.0	- 2.836	1.47
14.0	+ 40.0	- 1.836	3.96
15.0	+ 50.0	- 0.836	10.59
16.0	+ 60.0	+ 0.164	29.45
17.0	+ 70	+ 1.164	80.75

e. Machine Equations

$$\frac{d\eta'}{dt'} = 0.0697\, K'_{1ex} + 0.3512\,(0.0331\psi'_1 + 0.2199\psi'_2$$

$$+ 0.2821\psi'_3 + 0.3192\psi'_4 + 0.1126\psi'_5 + 0.0331\psi'_6)$$

$$\frac{d\psi'_1}{dt'} = -\frac{d\eta'}{dt'} - 0.0013\psi'_1 - 0.5\psi'_1 \frac{d\eta'}{dt'}$$

$$\frac{d\psi'_2}{dt'} = -\frac{d\eta'}{dt'} - 0.0032\psi'_2 - 0.5\psi'_2 \frac{d\eta'}{dt'}$$

$$\frac{d\psi'_3}{dt'} = -\frac{d\eta'}{dt'} - 0.0154\psi'_3 - 0.5\psi'_3 \frac{d\eta'}{dt'}$$

$$\frac{d\psi'_4}{dt'} = -\frac{d\eta'}{dt'} - 0.0456\psi'_4 - 0.5\psi'_4 \frac{d\eta'}{dt'}$$

$$\frac{d\psi_5'}{dt'} = 0.0001\,K_{1ex}' \frac{d\eta'}{dt'} - 0.1612\psi_5' - 0.5\psi_5' \frac{d\eta'}{dt'}$$

$$\frac{d\psi_6'}{dt'} = 0.0009\,K_{1ex}' \frac{d\eta'}{dt'} - 1.43\psi_6' - 0.5\psi_6' \frac{d\eta'}{dt'}$$

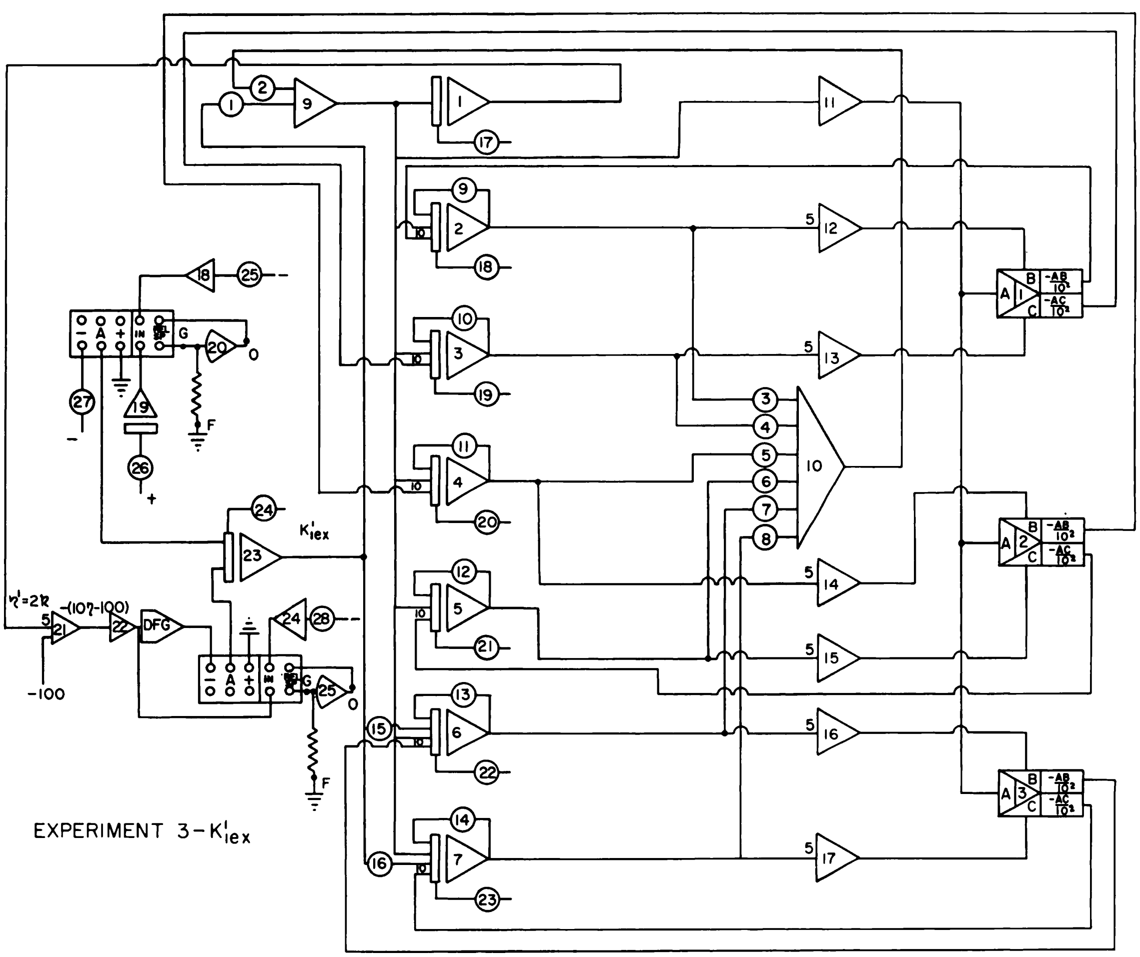
$$K_{1ex}' = K_1'(t) - K_1'(n,t)$$

$$K_1'(t) = \begin{cases} 13.25\,t' \text{ volts for } 0 \leq t' \leq 5 \text{ sec} \\ 66.23 \text{ volts for } t' > 5 \text{ sec} \end{cases}$$

$$K_1'(n,t) = 25 \exp(\eta - 15.836)$$

4. Analog Circuit Diagram

a. Flow Sheet



EXPERIMENT 3 − η = ln n₁

Fig. 8. Circuit Diagram for Duplication of TREAT Transient

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b. **POTENTIOMETER SETTINGS**

PROBLEM NO. ____________
DRAWING NO. ____________
DATE ____________

THE TREAT REACTOR

POTENTIOMETER NO. DRAWING	POTENTIOMETER NO. MACHINE	MATHEMATICAL VALUE	VALUE	CORRECTION	SETTING	SET	PARAMETERS
1		$\beta b(1-\beta)/a\ell c$	0.0697		0697		$\ell = 8.6 \times 10^{-4}$
2		$\beta/a\ell$	0.8779		8779		$\beta = 0.00755$
3		$b\beta_1/d_1\beta$	0.0331		0331		$\lambda_1 = 0.01246$
4		$b\beta_2/d_2\beta$	0.2199		2199		$\lambda_2 = 0.0315$
5		$b\beta_3/d_3\beta$	0.2821		2821		$\lambda_3 = 0.1535$
6		$b\beta_4/d_4\beta$	0.3192		3192		$\lambda_4 = 0.456$
7		$b\beta_5/d_5\beta$	0.1126		1126		$\lambda_5 = 1.612$
8		$b\beta_6/d_6\beta$	0.0331		0331		$\lambda_6 = 14.3$
9		λ_1/a	0.0012		0012		$\beta_1 = 0.00025$
10		λ_2/a	0.0032		0032		$\beta_2 = 0.00166$
11		λ_3/a	0.0154		0154		$\beta_3 = 0.00213$
12		λ_4/a	0.0456		0456		$\beta_4 = 0.00241$
13		λ_5/a	0.1612		1612		$\beta_5 = 0.00085$
14		λ_6/a	1.43		1430	(10)	$\beta_6 = 0.00025$
15		$d_5\lambda_5\beta/ac$	0.0001		0001		$a = 10$
16		$d_6\lambda_6\beta/ac$	0.0009		0009		$b = 2$
17		FOR	0				$c = 25$
TO		STATIC	0				$d_1 = \ldots = d_6 = 2$
24		CHECK	0				$n(0) = 10^2$
25		- 5.0 volts			0500		
26		+1.0 volts			0100		
27		$-0.04c/\beta a$ volt			1325		
28		-19.88 volts			1988		

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C. STATIC CHECK

PROBLEM NO.
DRAWING NO.
DATE

THE TREAT REACTOR

UNIT	UNIT NUMBER DRAWING	UNIT NUMBER MACHINE	OUTPUT (VOLTS)	REMARKS	INTEGRATOR	INITIAL CONDITION	SET	PARAMETERS
POT	17		-10		1	+10		
	18		+10		2	-10		
	19		+10		3	-10		
	20		+10		4	-10		
	21		+10		5	-10		
	22		+10		6	-10		
	23		+10		7	-10		
	24		-50		23	+50		
AMP	9		-7.0					
	10		-10					
	11		-10					
	12		+50					
	13		+50					
	14		+50					
	15		+50					
	16		+50					
	17		+50					
	18		+5.0					
	20		Negative					
	25		Positive					
	ALL MULTIPLIER CHANNELS = +5 volts							

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5. Graphical Results

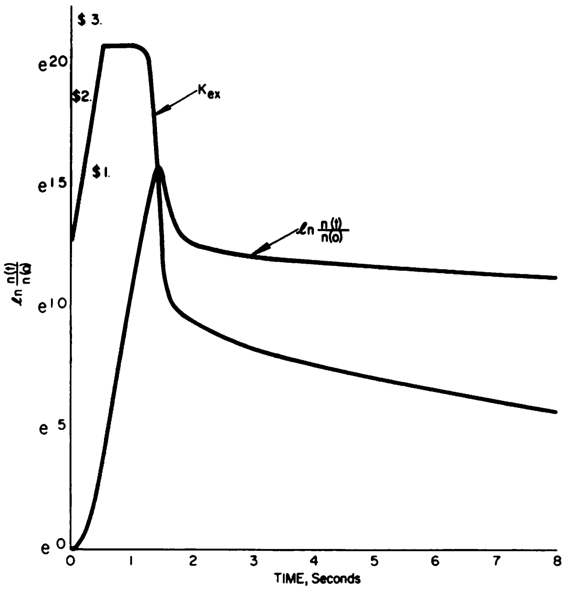

Fig. 9

K_{ex} and $\eta = \ell n\ n$, Versus Time

6. Bibliography

III-1. Okrent, D., The Reactor Kinetics of the Transient Reactor Test Facility (TREAT), ANL-6174 (Sept. 1960)

III-2. Bryant, L. T., Just, L. C., and Pawlicki, G. S., Introduction To Electronic Analog Computing, ANL-6187 (July 1960).

III-3. Scott, W. E., Fundamental Components of the "PACE" Analog Computer, ANL-6075 (Nov. 1959).

IV. A VIBRATING SYSTEM WITH TWO DEGREES OF FREEDOM

1. Problem Description

Problems of vibration must be considered in the design of power plants using fissionable fuel. Fuel elements, control rods, and structural supporting members are capable of vibrating; their characteristics must be analyzed, for vibration problems prove to be of importance to eliminate concern for the safe operation of the power plant. Good representations of the true situation usually involve systems with several degrees of freedom.(IV-1)

A typical vibration problem which will serve as an introduction to multi-degree-of-freedom systems is shown in Fig. 10. The two masses m_1 and m_2 are suspended vertically by springs k_1 and k_2. The masses are constrained such that they only move vertically. The displacements x_1 and x_2, taken positive for a downward motion, are measured using static equilibrium as reference. The elongation of the upper spring is x_1 and the elongation of the lower spring is $(x_2 - x_1)$. The restoring force acting on m_1 is $[-k_1x_1 + k_2(x_2 - x_1)]$, and on m_2 the restoring force is $-k_2(x_2 - x_1)$, where k_1 and k_2 are the spring constants of the respective springs.

Effects due to energy dissipation in the elastic spring, wind friction, and springs that have appreciable mass have been neglected in the equations of motion given below.

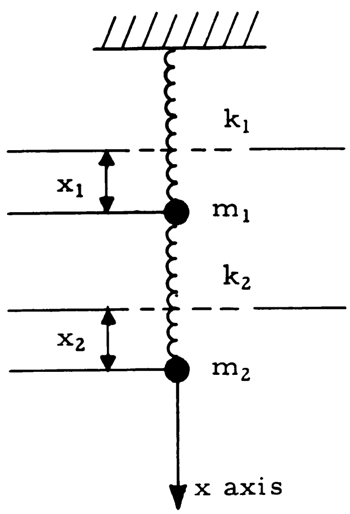

Fig. 10

Illustration of a Vibrating System with Two Degrees of Freedom

2. Mathematical Statement of the Problem.

a. Equations

$$m_1 \frac{d^2x_1}{dt^2} = -k_1x_1 + k_2(x_2 - x_1) \qquad (1)$$

$$m_2 \frac{d^2x_2}{dt^2} = -k_2(x_2 - x_1) \qquad (2)$$

b. Equation Constants

m_i: mass (lb) (i = 1, 2)

k_i: spring constant (lb force/ft) (i = 1, 2)

A: Initial displacement of the springs (feet)

c. Initial Conditions

It is obvious that with the springs displaced a certain distance, A, the initial conditions will have the following values

$$x_1(0) = x_2(0) = A$$

$$\frac{dx_1(0)}{dt} = \frac{dx_2(0)}{dt} = 0 \qquad (3)$$

$$\frac{d^2x_1(0)}{dt^2} = \frac{d^2x_2(0)}{dt^2} = 0$$

3. Preparation of Machine Equations:

In transforming to the machine equations, the following relationships are made. Let

$$x_i' = bx_i \quad (i = 1, 2)$$

and (4)

$$t' = at \quad .$$

Substitution of equations (4) into equations (1) and (2) yields

$$\frac{d^2x_1'}{dt'^2} = - \frac{k_1}{a^2m_1} x_1' + \frac{k_2}{a^2m_1} (x_2' - x_1') \qquad (5)$$

$$\frac{d^2x_2'}{dt'^2} = \frac{-k_2}{a^2m_2} (x_2' - x_1') \qquad (6)$$

where

$$x_1'(0) = bx_1(0) = bA$$

and

$$x_2'(0) = bx_2(0) = bA \quad .$$

The solution to the equations will vary with m_1, m_2, k_1, k_2, and A. In the solution given here, we consider the following physical constants:

$$k_1 = k_2 = 0.2 \text{ lb force/ft}$$

$$m_1 = m_2 = 1 \text{ lb mass}$$

$$A = 1 \text{ ft}$$

4. Analog Circuit Diagram

a. Flow Sheet

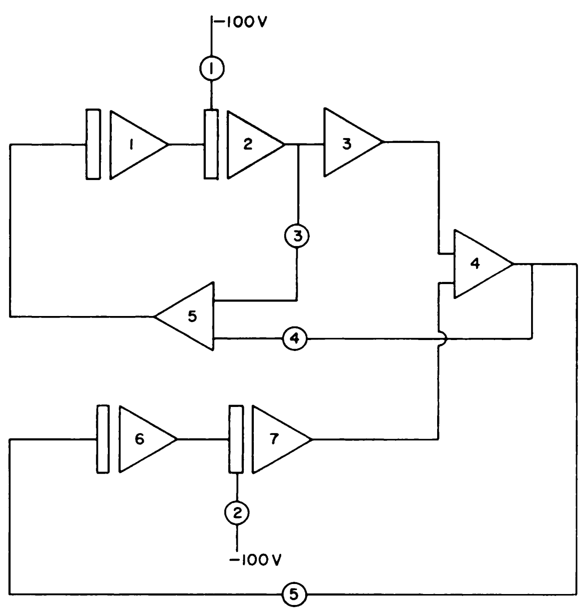

Fig. 11. Circuit Diagram for the Solution of the Equations Describing a Vibrating System with Two Degrees of Freedom

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b. **POTENTIOMETER SETTINGS**

PROBLEM NO. ______
DRAWING NO. ______
DATE ______

A VIBRATION SYSTEM WITH
TWO DEGREES OF FREEDOM

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORRECTION	SETTING	SET	PARAMETERS	
DRAWING	MACHINE							
1		bA	-50.00		-5000			a = 1
2		bA	-5000		-5000			b = 50
3		k_1/a^2m_1	0.2		2000			
4		k_2/a^2m_1	0.2		2000			
5		k_2/a^2m_2	0.2		2000			

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c. **STATIC CHECK**

PROBLEM NO. ______
DRAWING NO. ______
DATE ______

UNIT	UNIT NUMBER		OUTPUT (VOLTS)	REMARKS	INTEGRATOR	INITIAL CONDITION	SET	PARAMETERS
	DRAWING	MACHINE						
POT	3		10.00					
	4		0.0					
	5		0.0					
AMP	1		0.0					
	2		+50.0					
	3		-50.0					
	4		0.0					
	5		-10.0					
	6		0.0					
	7		50.0					

5. Graphical Results

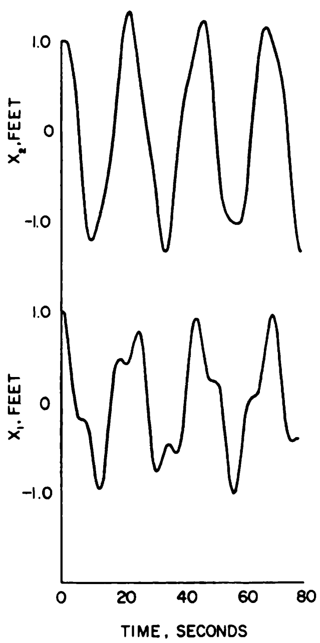

Fig. 12

Distance Versus Time for a System with Two Degrees of Freedom

6. Bibliography

IV-1. Timoshenko, S., Vibration Problems in Engineering, D. Van Nostrand Company, Inc., Princeton, New Jersey, (1955) 3rd. Ed.

V. TEMPERATURE DISTRIBUTION IN A RADIATING FIN

1. Problem Description

The only economical method for rejecting heat from an outer-space power plant is by thermal radiation.(V-1) If the working fluid of the power plant passes through tubes, the additions of extended surfaces to the tubes in the form of fins reduces the number of tubes required. This reduction decreases the probability that a meteor will puncture a vital coolant-carrying passageway. (The puncture of a fin is of lesser concern for the continued operation of the power plant.) An analysis of the temperature distribution of these extended surfaces is very important in calculating the effectiveness (and, indirectly, the safety of the plant) of various fin geometries.

2. Mathematical Statement of the Problem

In the development of a differential equation for conduction,(V-2)

$$dq = d/dx \left(2kWY_x \frac{dT}{dx} dx \right) \quad . \tag{1}$$

A general heat balance requires this differential equation (1) to be equal to

$$dq = 2\sigma\epsilon \, (T^4 - T_s^4) \, dA \quad , \tag{2}$$

the heat rejected by radiation.

By assuming that the arc length (ds) on the arbitrary surface is equivalent to dx on the abscissa and assigning Y_x equal to a constant thickness for a straight fin geometry, the differential equation for temperature is

$$\frac{d^2T}{dx^2} = \frac{\sigma\epsilon}{Hk} \, (T^4 - T_s^4) \quad . \tag{3}$$

A constant heat source will be assumed at one end of the fin and $\left.\frac{dT}{dx}\right|_{x=L} = 0$ at the other end.(V-3) This will correspond to the fin in Fig. 13.

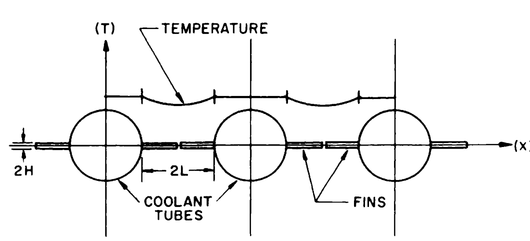

Fig. 13

Geometry of Radiation Fin and Coolant Tubes

a. Constants and Variables

T = Absolute temperature along the fin

T_s = Absolute temperature of the sink

σ = Stefan-Boltzmann constant

ϵ = Emissivity

W = Width of the fin in the z-direction

L = Total length of the fin in the x-direction

q = Heat dissipated

H = Half-thickness of the fin

k = Thermal conductivity of the fin material

Typical values are:

$T_s = 0°R$

$\sigma = 0.173 \times 10^{-8}$ BTU/(hr)(ft^2)(°R^4)

$\epsilon = 0.9$

$W = 1.0$ ft

$L = 0.25$ ft

$H = 1.250 \times 10^{-3}$ ft

$k = 25.0$ BTU/(hr)(ft)(°R)

b. Initial Conditions

$T(0) = 2000°R$

The most efficient use of radiator material weight dictates the arrangement of the finned tubes in a straight bank. The general temperature distribution, of this arrangement, along the fin is given in Fig. 15.

3. Preparation of Machine Equations

a. Machine Variables and Scale Factors.

$$
\begin{aligned}
ax &= x' & bT &= T' \\
adx &= dx' & bdT &= dT' \\
a^2dx^2 &= dx'^2 & bd^2T &= d^2T' \\
a &= 10^2 & b &= 5 \times 10^{-2}
\end{aligned}
\qquad (5)
$$

b. Scaled Equation

Substituting Equations (5) into Equation (3) we get,

$$\frac{d^2T'}{dt'^2} = \frac{\sigma\epsilon T'^4}{a^2b^3kH} \quad . \qquad \text{(Note } T_s = 0.)$$

c. Machine Equation

$$\frac{d^2T'}{dt'^2} = 0.03986 \left(\frac{T'^4}{10^6}\right)$$

d. Initial Conditions

$T' = bT = 100$ volts

$$\frac{dT'}{dt'} = Y \text{ volts}$$

so that

$$\left.\frac{dT'}{dt'}\right|_{t=aL} = 0$$

4. Analog Circuit Diagram

a. Flow Sheet

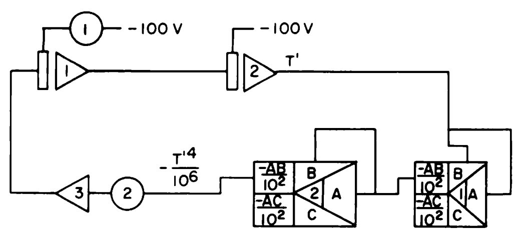

Fig. 14. Circuit Diagram for Solution of Second-order, Fourth-degree Differential Equation

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b. **POTENTIOMETER SETTINGS**

PROBLEM NO. ____
DRAWING NO. ____
DATE ____

POTENTIOMETER NO. DRAWING	POTENTIOMETER NO. MACHINE	MATHEMATICAL VALUE	VALUE	CORRECTION	SETTING	SET	PARAMETERS
1		$-\frac{dT'}{dt'}$ volts	*				
2		$10^6\sigma\epsilon/a^2b^3kH$	0.03986		0399		
*	Variable to produce	$\frac{dT}{dx}=$	0 at x	= L			

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c. **STATIC CHECK**

PROBLEM NO. ____
DRAWING NO. ____
DATE ____

UNIT	UNIT NUMBER DRAWING	UNIT NUMBER MACHINE	OUTPUT (VOLTS)	REMARKS	INTEGRATOR	INITIAL CONDITION	SET	PARAMETERS
POT	2		-3.99		2	+100		
MUH	1 AB		-100v					
	2 AB		-100v					
AMP	3		+3.99					

5. Graphical Results

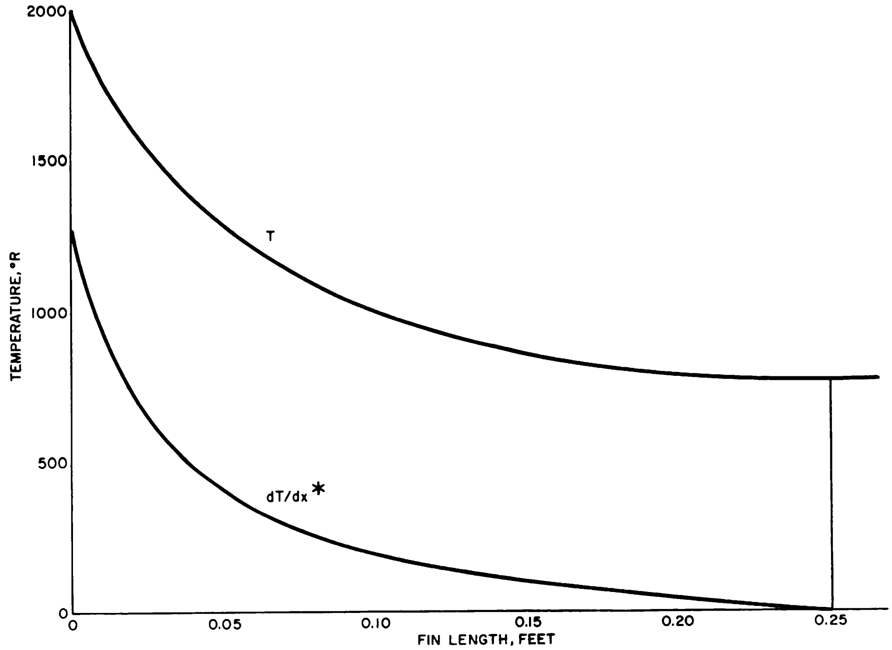

Fig. 15. Temperature Versus Length and dT/dx Versus Length for a 0.25-ft Fin (K = 25.0)

*dT/dx was plotted to show when boundary condition $\left.\frac{dT}{dx}\right|_{x=L} = 0$ is satisfied.

6. Bibliography

V-1. Corliss, W. R., Nuclear Power in Outer Space, Nucleonics, 18 (8), 59-63 (1960).

V-2. Schneider, P. J., Conduction Heat Transfer, Addison - Wesley Publishing Company, Inc., Reading Massachusetts (1955).

V-3. Lieblein, S., Analysis of Temperature Distribution and Radiant Heat Transfer Along a Rectangular Fin of Constant Thickness, NASA Technical Note D-196. National Aeronautics and Space Administration, Washington (November 1959).

V-4. Carslaw, H. S., and Jaeger, J. C., Conduction of Heat in Solids, Second Edition, Oxford University Press, Amen House, London E. C. 4, 1959.

VI. TEMPERATURE DISTRIBUTION IN AN INFINITE SLAB CONSIDERING VARIABLE THERMAL PROPERTIES

1. Problem Description

When the thermal properties of various materials are studied, thermal conductivity, specific heat and density are usually considered as constants; they are, however, dependent upon temperature.(VI-1) In this experiment, an insulated zirconium slab is studied. Four cases are considered:

(1) Diffusivity ($\kappa = k/\rho c$) is constant;

(2) $\kappa = F\left(\frac{1}{5}\sum_{i=1}^{5} T_i\right)$;

(3) κ = F (Temperature of the region described by the heat balance);

(4) κ = F (Average temperature across an interface).

2. Mathematical Statement of the Problem(VI-2)

a. Equations

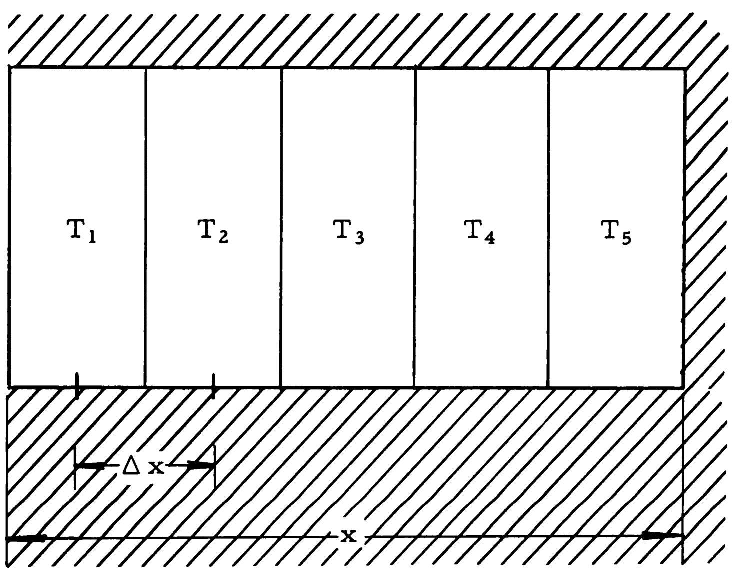

Fig. 16. Model of the Infinite Slab Showing Regions Used for Analysis

$$\frac{dT_1}{dt} = \frac{S}{\rho c \Delta x} - \frac{\kappa}{\Delta x^2}(T_1 - T_2)$$

$$\frac{dT_2}{dt} = \frac{\kappa}{\Delta x^2}(T_1 - T_2) - \frac{\kappa}{\Delta x^2}(T_2 - T_3)$$

$$\frac{dT_3}{dt} = \frac{\kappa}{\Delta x^2}(T_2 - T_3) - \frac{\kappa}{\Delta x^2}(T_3 - T_4)$$

$$\frac{dT_4}{dt} = \frac{\kappa}{\Delta x^2}(T_3 - T_4) - \frac{\kappa}{\Delta x^2}(T_4 - T_5)$$

$$\frac{dT_5}{dt} = \frac{\kappa}{\Delta x^2}(T_4 - T_5) - \frac{\epsilon\sigma}{\rho c \Delta x}(T_5^4 - T_0^4).*$$

b. Constants

(1) Constant case

k = thermal conductivity = 11 BTU/(hr)(ft)(°F)

c = specific heat = 0.066 BTU/(lb)(°F)

ρ = density = 0.397 lb/ft^3

$\kappa = k/c\rho$ = diffusivity = 0.4198 ft^2/hr

S = heat source = 18.3 BTU/(ft^2)(sec)

ϵ = emissivity

σ = Stephan-Boltzmann constant

$\Delta x = 1/60$ ft

(2) As a function of temperature

T, °F	κ(T)
100	0.4198
200	0.4
300	0.38
400	0.364
500	0.352
600	0.336
700	0.322
800	0.309
900	0.298

*Radiation heat loss will be neglected.

c. Initial Conditions

$T_1 = T_2 = T_3 = T_4 = T_5 = 100°F$.

3. Preparation of Machine Equations

a. Machine Variables

$t' = at$

$T' = bT$

$S'' = \Delta x S b / k$

b. Scale Factors

$a = 1.0$

$b = 0.1$

c. Machine Equations

$$\frac{dT_1'}{dt'} = \frac{\kappa S''}{\Delta x^2 a} - \frac{\kappa}{\Delta x^2 a}(T_1' - T_2')$$

$$\frac{dT_2'}{dt'} = \frac{\kappa}{\Delta x^2 a}(T_1' - T_2') - \frac{\kappa}{\Delta x^2 a}(T_2' - T_3')$$

$$\frac{dT_3'}{dt'} = \frac{\kappa}{\Delta x^2 a}(T_2' - T_3') - \frac{\kappa}{\Delta x^2 a}(T_3' - T_4')$$

$$\frac{dT_4'}{dt'} = \frac{\kappa}{\Delta x^2 a}(T_3' - T_4') - \frac{\kappa}{\Delta x^2 a}(T_4' - T_5')$$

$$\frac{dT_5'}{dt'} = \frac{\kappa}{\Delta x^2 a}(T_4' - T_5')$$

d. Initial Conditions

$T' = T_2' = \ldots T_5' = 10$ volts

$S'' = 10$ volts [$S = 18.3$ BTU/(ft^2)(sec)] for 50 sec

4. Analog Circuit Diagrams

a. Flow Sheets

Case I - (κ = constant)

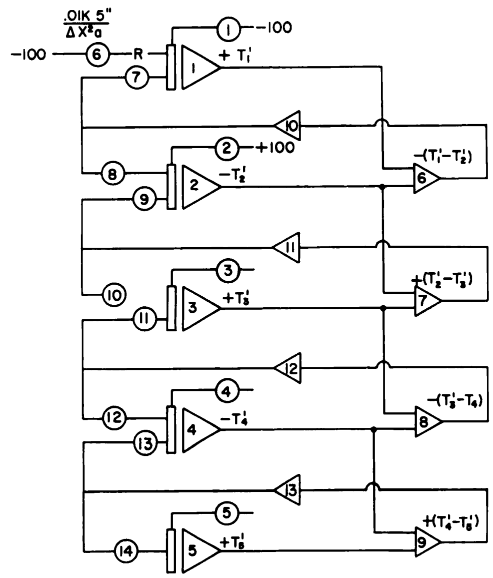

Fig. 17. Circuit Diagram for an Infinite Slab with Thermal Conductivity κ = Constant

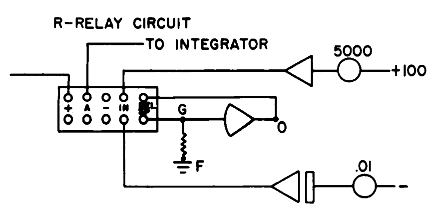

Fig. 18. Relay Circuit for the Heat Pulse Used in Experiment VI

Case II - [$\kappa = F(T_{av})$]

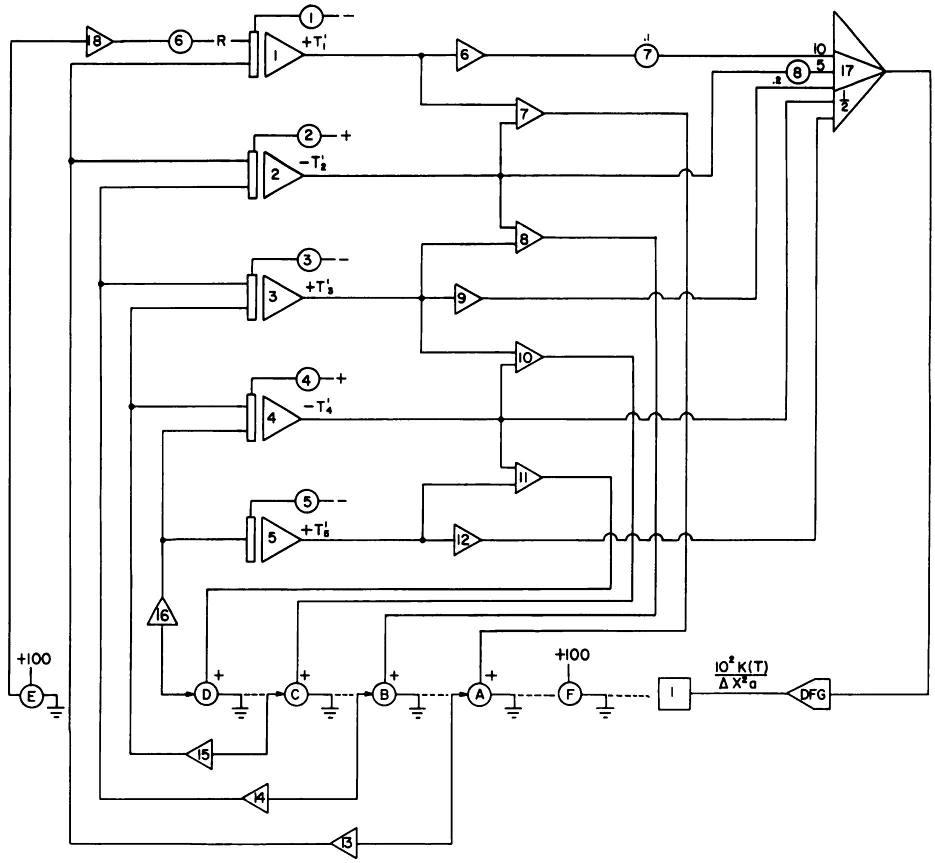

Fig. 19. Circuit Diagram for an Infinite Slab with Thermal Conductivity $\kappa = F(T_{average})$

Case III - [κ = F(Temperature of the Region Described by the Heat Balance)]

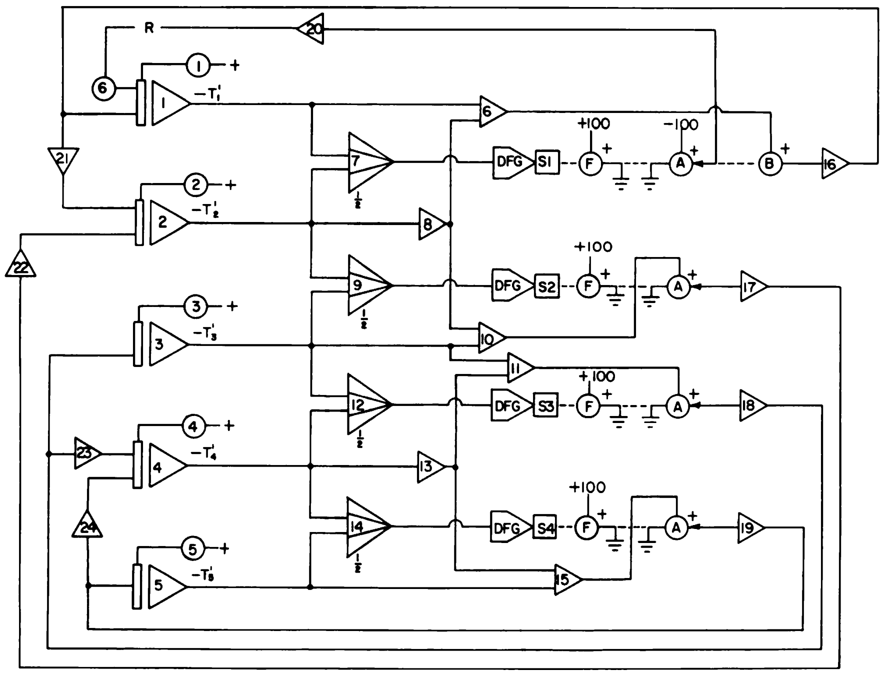

Fig. 20. Circuit Diagram for an Infinite Slab with Thermal Conductivity κ = F(Temperature of Region Described by the Heat Balance)

Case IV - [κ = F(Average Temperature Across an Interface)]

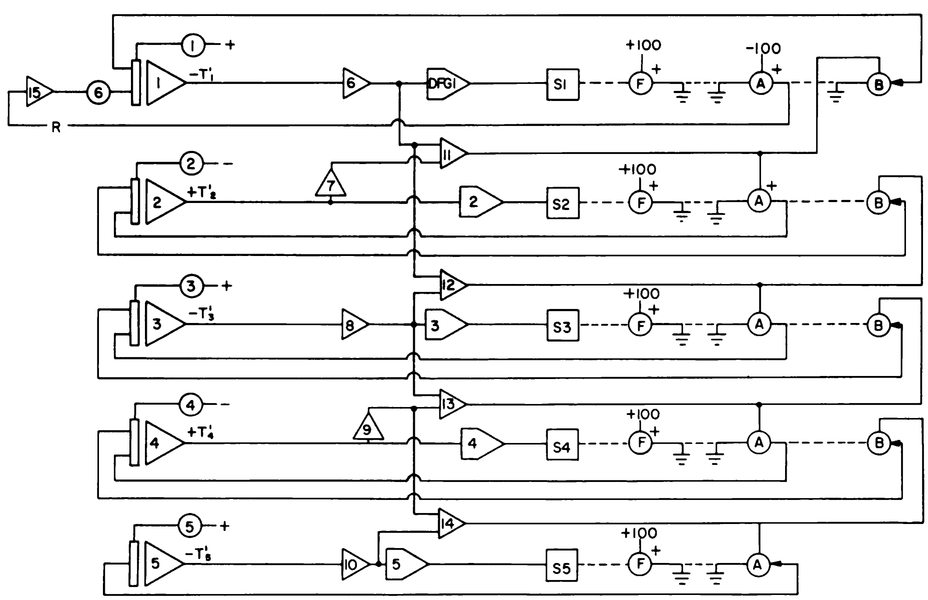

Fig. 21. Circuit Diagram for an Infinite Slab with Thermal Conductivity κ = F(Average Temperature Across an Interface)

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b. **POTENTIOMETER SETTINGS**

TEMPERATURE VARIATION IN A ONE FACE INSULATED SLAB
CONSIDERING VARIABLE THERMAL PROPERTIES
CASE I

PROBLEM NO. ________
DRAWING NO. ________
DATE ________

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORRECTION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
1		$0.01T_1'$	0.1		1000		S = 18.3
2		$0.01T_2'$	0.1		1000		S" = 10 volts
3		$0.01T_3'$	0.1		1000		a = 1
4		$0.01T_4'$	0.1		1000		b = 0.1
5		$0.01T_5'$	0.1		1000		$\kappa = \frac{0.4198}{3600}$
6		$(0.01\kappa S''/\Delta x^2 a)$	0.042		0420		$\Delta x = 1/60$
7		$\kappa/\Delta x^2 a$	0.4198		4198		$T_i(0) = 100°$
8		$\kappa/\Delta x^2 a$	0.4198		4198		
9		$\kappa/\Delta x^2 a$	0.4198		4198		
10		$\kappa/\Delta x^2 a$	0.4198		4198		
11		$\kappa/\Delta x^2 a$	0.4198		4198		
12		$\kappa/\Delta x^2 a$	0.4198		4198		
13		$\kappa/\Delta x^2 a$	0.4198		4198		
14		$\kappa/\Delta x^2 a$	0.4198		4198		
			CASE II				
1		$0.01T_1'$	0.1		1000		
2		$0.01T_2'$	0.1		1000		
3		$0.01T_3'$	0.1		1000		
4		$0.01T_4'$	0.1		1000		
5		$0.01T_5'$	0.1		1000		
6		S"/100	0.1		1000		
7		1.0	1.0		1000	(10)	
8		1.0	1.0		2000	(5)	

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b. **POTENTIOMETER SETTINGS**

TEMPERATURE VARIATION IN A ONE FACE INSULATED SLAB
CONSIDERING VARIABLE THERMAL PROPERTIES
CASE III - IV

PROBLEM NO. ______
DRAWING NO. ______
DATE ______

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC-TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
1		$0.01T'_1$	0.1		1000		
2		$0.01T'_2$	0.1		1000		
3		$0.01T'_3$	0.1		1000		
4		$0.01T'_4$	0.1		1000		
5		$0.01T'_5$	0.1		1000		
6		$S''/100$	0.1		1000		

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c. STATIC CHECK

CASE I

PROBLEM NO.
DRAWING NO.
DATE

UNIT	UNIT NUMBER		OUTPUT (VOLTS)	REMARKS	INTE-GRATOR	INITIAL CONDI-TION	SET	PARAMETERS
	DRAWING	MACHINE						
AMP	6		0.0		1	+10		
	7		0.0		2	-10		
	8		0.0		3	+10		
	9		0.0		4	-10		
POT	6		-4.2		5	+10		
	7to14		0.0					
				CASE II				
AMP	6		-10		1	+10		
	7		0		2	-10		
	8		0		3	+10		
	9		-10		4	-10		
	10		0		5	+10		
	11		0					
	12		-10					
	13to16		0					
	17		10					
	18		-41.98					
DFG			+41.98					
POT	6		-4.2					
	7		-1.0					
	8		-2.0					

AMD-2A (8-57)

Argonne National Laboratory

APPLIED MATHEMATICS DIVISION
ANALOG COMPUTER

c. **STATIC CHECK**

PROBLEM NO.
DRAWING NO.
DATE

TEMPERATURE VARIATION IN A ONE FACE INSULATED SLAB
CONSIDERING VARIABLE THERMAL PROPERTIES
CASE III

UNIT	UNIT NUMBER		OUTPUT (VOLTS)	REMARKS	INTE-GRATOR	INITIAL CONDI-TION	SET	PARAMETERS
	DRAWING	MACHINE						
AMP	11		0		1	-10.0		
	12		0		2	+10		
	13		0		3	-10		
	14		0		4	+10		
	15		+41.98		5	-10		
DFG	1to5		+41.98					
POT	6		+4.2					
				CASE IV				
AMP	7		+10		1	-10.0		
	9		+10		2	-10.0		
	12		+10		3	-10.0		
	14		+10		4	-10.0		
	6		0.0		5	-10.0		
	10		0.0					
	11		0.0					
	15		0.0					
	16to24		0.0					
	20		+41.98					
	6		+4.2					

AMD-2A (8-57)

5. Graphical Results

Case I - (κ = constant)

Heat Input = 18.3 BTU/ft^{2}sec for 50 sec

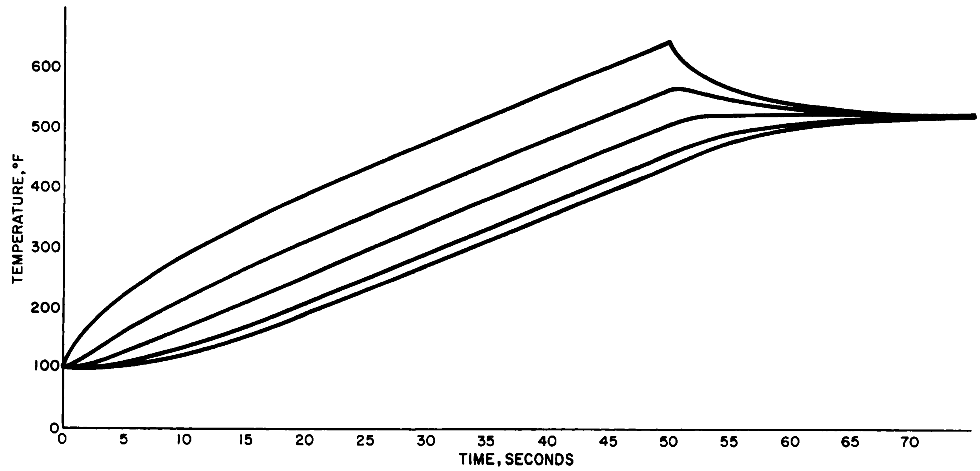


Fig. 22. Temperature Distribution for an Infinite Slab - Case I

Case II - [$\kappa = F(T_{av})$]

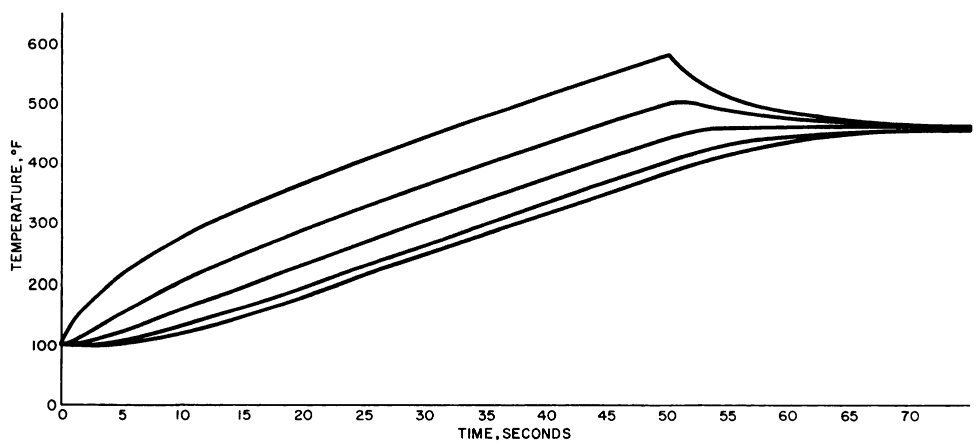


Fig. 23. Temperature Distribution for an Infinite Slab - Case II

Case III - [κ = F(T of region described by the heat balance)]

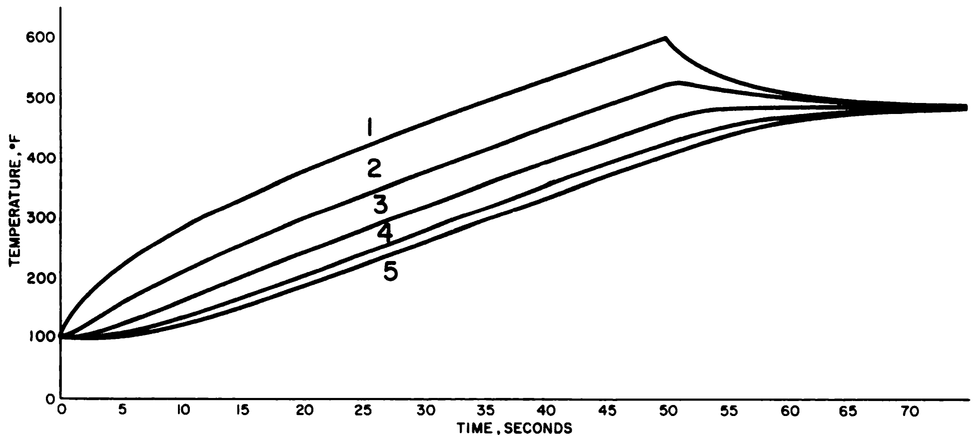


Fig. 24. Temperature Distribution for an Infinite Slab - Case III

Case IV - [κ = F(Average T across an interface)]

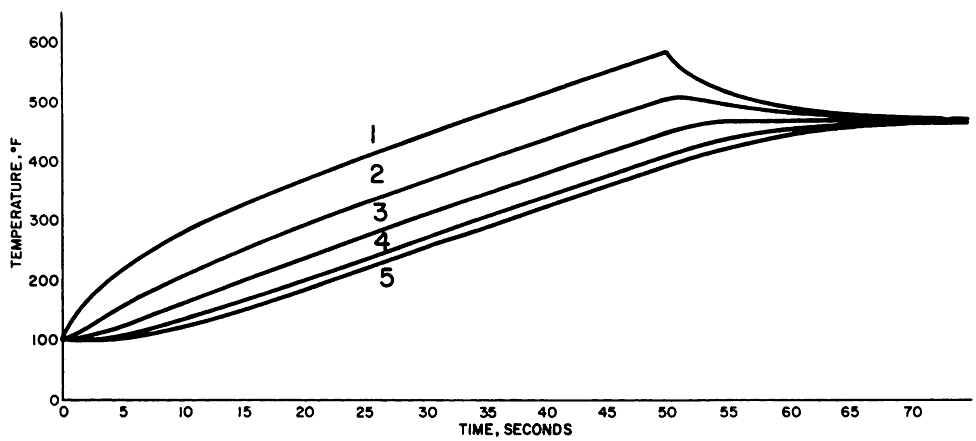


Fig. 25. Temperature Distribution for an Infinite Slab - Case IV

6. Bibliography

VI-1. Tebo, F. J., Selected Values of the Physical Properties of Various Materials, ANL-5914, (September 1958).

VI-2. Taraba, F. R., Personal Communication.

VII. IODINE-XENON BUILDUP IN A REACTOR

1. Problem Description

The time-dependent behavior of iodine and xenon will be studied under the following conditions:

(1) constant flux;
(2) step change in flux; and
(3) sinusoidal flux variation.

In addition, the effect of fuel depletion upon the iodine and xenon concentrations can be studied.

2. Mathematical Statement of the Problem

a. Equations

$$\frac{dI}{dt} = \gamma_I \sigma_{fU} U \phi - \lambda_I I$$

$$\frac{dX}{dt} = \gamma_X \sigma_{fU} U \phi + \lambda_I I - \lambda_X X - \sigma_{aX} X \phi$$

$$\frac{dU}{dt} = - \sigma_{aU} \phi U$$

b. Range of Variables

$$0 \leq \phi \leq 1.5 \times 10^{14} \text{ n/cm}^2\text{-sec}$$

$$0 \leq U \leq 6.4 \times 10^{20} \text{ a/cm}^3$$

c. Constants

$\lambda_I = 0.1033/3600$

$\lambda_X = 0.0752/3600$

$\gamma_I = 0.064$

$\gamma_X = 0.003$

$\sigma_{aX} = 3.0 \times 10^{-18}$

$\sigma_{fU} = 3.98 \times 10^{-22}$

$\sigma_{aU} = 4.72 \times 10^{-22}$

d. Initial Conditions

$I(0) = 0$

$X(0) = 0$

$U(0) = 6.4 \times 10^{20}$

$\phi(0) = 1.5 \times 10^{14}$

3. Preparation of Machine Equations

a. Machine Variables

$t' = a_1 t$	$a_1 = 1/3600$
$U' = a_2 U$	$a_2 = 10^{-19}$
$\phi' = a_3 \phi$	$a_3 = 10^{-13}$
$I' = a_4 I$	$a_4 = 10^{-15}$
$X' = a_5 X$	$a_5 = 10^{-15}$

b. Scaled Equations

$$\frac{dI'}{dt'} = \frac{a_4 \sigma_{fU} \gamma_I}{a_1 a_2 a_3} U'\phi' - \frac{\lambda_I}{a_1} I'$$

$$\frac{dX'}{dt'} = \frac{a_5 \sigma_{fU} \gamma_X}{a_1 a_2 a_3} U'\phi' + \frac{\lambda_I a_5 I'}{a_1 a_4} - \frac{\lambda_X X'}{a_1} - \frac{\sigma_{aX}}{a_1 a_3} X'\phi'$$

$$\frac{dU'}{dt'} = - \frac{\sigma_{aU}}{a_1 a_3} \phi' U'$$

c. Machine Equations

$$\frac{dI'}{dt'} = 0.917 \left(\frac{U'\phi'}{10^2}\right) - 0.1033 I'$$

$$\frac{dX'}{dt'} = 0.0430 \left(\frac{U'\phi'}{10^2}\right) + 0.1033 I' - 0.0752 X' - 1.08 \left(\frac{X'\phi'}{10}\right)$$

d. Initial Voltages

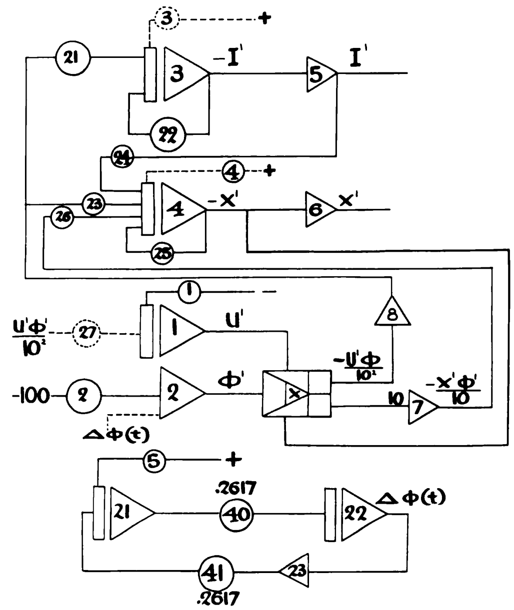
$U' = 64$ v

$\phi' = 15$ v

$I' = 0$

$X' = 0$

4. Analog Computer Circuit



5. Potentiometer Settings

No.	Math. Value	Num. Value	Setting
21	$\frac{a_4 \sigma_{fU} \gamma_I 10^2}{a_1 a_2 a_3}$	0.916992	9170
22	λ_I / a_1	0.1033	1033
23	$\frac{a_5 \sigma_{fU} \gamma_X 10^2}{a_1 a_2 a_3}$	0.042984	0430
24	$a_5 \lambda_I / a_1 a_4$	0.1033	1033
25	λ_X / a_1	0.0752	0752
26	$\frac{10 \sigma_{aX}}{a_1 a_3}$	1.08	1080(10)
1	U'(0)	64.00	6400
2	ϕ'(0)	15.00	1500
3, 4	Steady state value for I' & X' if experiment is started from $t \neq 0$		
27	Optional: used if fuel is to burn-up $\frac{\sigma_{aU} 10^2}{a_1 a_3}$	0.001699	0017
5, 40, 41	Used to regulate a sinusoidal change in flux (period of 1 day)		