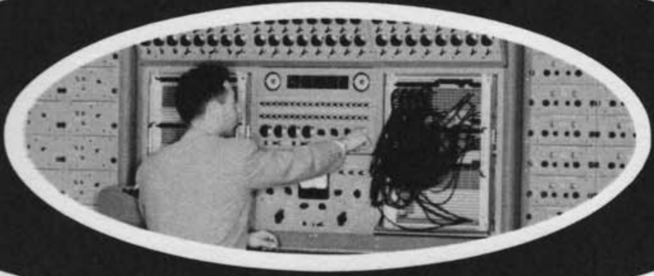
APPLICATION BULLETIN No. 1

Process Control Problems Yield to the Analog Computer



Analog Computer

APPLICATION BULLETIN

The **Key** *to Progressive Engineering*

Electronic Associates, Inc.

Manufacturers of PACE *Precision Analog Computing Equipment*

BULLETIN No. AB-856

Process-Control Problems Yield to the Analog Computer

The analog computer has given systems engineering a big boost over the older methods of design for process control systems in the chemical industry. Interestingly, the answer is still "cut-and-try", but the cut-and-try is now done in minutes by turning computer knobs instead of in months by changing actual control elements on the process. The savings in time and money are obvious. This article shows, step by step, how a batch chemical process and a control system proposed for it were set up and solved by simulation on an analog computer. Though it did not happen in this case, the computer might have shown the need for a change in the process itself to attain satisfactory control with practical instruments.

C. W. WORLEY, Electronic Associates, Inc., and
R. W. E. FRANKS and J. F. PINK, E. I. du Pont de Nemours & Co., Inc.

Chemical processes often present control problems that are unwieldy by the usual methods of solution. Such a problem arose recently in a batch heating operation, and it was decided that simulation of the process and its control system on an analog computer would produce an accurate answer in the quickest and least-expensive way. This article tells, step by step, how this simulation was done. The process described here is a good one to illustrate the method because it includes several interesting simulation problems and because its related batch operation is common in the process industries.

Description of the physical system

Physically, the problem involves only the batchholder (a large kettle with a jacket into which steam can be injected to heat the batch), its instrumentation, and the control system. Temperature is the unwieldly parameter in this case because of the strict process requirements that it follow as closely as possible the ideal temperature-time curve of Figure 1. Specifically, the batch enters the kettle with its temperature somewhat below 150 deg F, is heated as quickly as possible to about 200 deg F, at which it is held for a fixed time, and then is quickly cooled and sent on. A variation of only 1 deg F can cause off-standard product and losses.

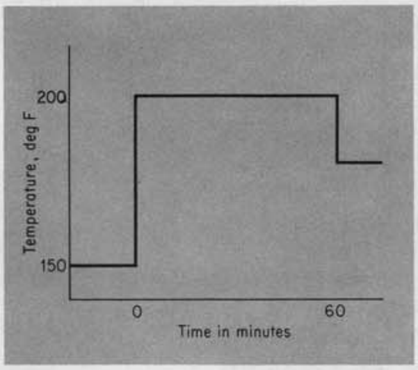


FIG. 1. Desired temperature response for batch process described.

The low temperature-overshoot required by the process makes adequate reproduction of the curve of Figure 1 impractical with manual controls. But automatic control introduces other problems. For one, the large thermal inertia of the batch controls the rate at which batch temperature rises after steam is admitted to the kettle jacket. For another, batch size can vary, which means a variable heat transfer lag. Also, the precise temperature to which the batch is heated can be changed according to process requirements, and steam and water supply pressures can fluctuate.

Several questions arise at this point which are answered most easily by an analog simulation. For example:

▶ Is it possible to attain reproducible start-up conditions for every batch, with a maximum transient overshoot of less than ½ deg F?

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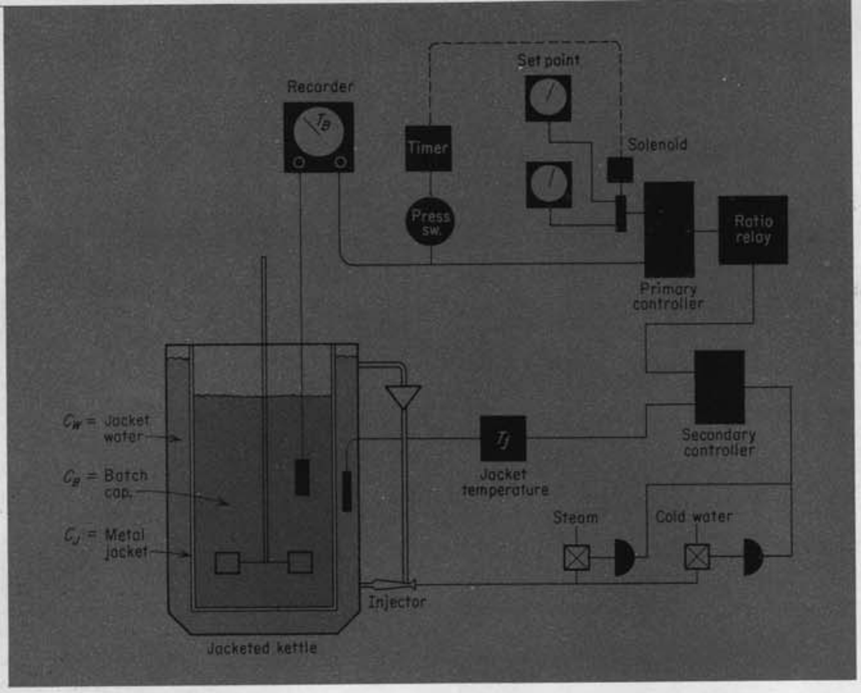


FIG. 2. Batch process and cascade-control systems.

- ▶ Is a cascade-control system superior to a single-loop control system? If so, by how much?
- ▶ What will be the optimum controller settings?
- ▶ How much will the system stability and reproducibility be affected by changes in steam pressure, batch size, ambient temperature, cooling water temperature, and batch reaction?
- ▶ Will controller drift be a serious problem in maintaining system response within the required limits?

The simulated system

Simulation of the batch operation with a cascade-control system will be described in detail in this article. Simulation of the single-loop system will not be described, since it is relatively simple.

In the cascade system, shown in Figure 2, the batch temperature is recorded and transmitted by the primary controller to adjust the set-point of the secondary controller. The secondary controller operates "dualled" valves to supply either cold water or steam to the jacket of the kettle. A ratio relay in the output of the primary controller limits the set-point signal applied to the secondary controller. Without this limit, the jacket wall would become excessively hot during warm-up when a large error exists, and thus would compromise product quality.

The primary measuring element in the batch is a resistance-bulb thermometer, chosen for its accuracy and reliability. It has a measured time constant of 4 sec. Its output is converted to a pneumatic signal and applied to the primary controller. This same pneumatic signal actuates a pressure switch that is preset to close when the holding temperature is reached. The pressure switch starts the timer that determines the holding period, and after this period the set-point of the primary controller is dropped to a lower temperature. This is done by a solenoid valve and two variable set-point pressure supplies.

Jacket temperature is measured by a capillary-type bulb placed within the jacket so as to measure an average temperature. Both controllers are pneumatic and have three adjustable modes—rate, automatic reset, and gain. Whether valves have equal percentage or linear trim would be determined by the analog study.

The computer solution

The usual reason for simulating a system by analogs is that the analog system is relatively inexpensive and quick to change, and obeys the same differential equations for its behavior in time as the original system. The early mechanical analogs have given way to the modern electronic analog computer primarily because the electronic analog elements are relatively noninteracting; i.e., they do not load one another and thereby cause changed dynamic characteristics. Electronic analogs also offer great flexibility in circuit arrangement, and are relatively inexpensive for high precisions. Although voltages at various points in the computer simulate forces, velocities, temperatures, etc., in the original system, the circuits also represent its differential equation; hence the term "computer".

Returning to the batch heating operation, the basic law of heat transfer by conduction can be expressed in the form of the rate of heat transfer:

$$\text{Rate} = \frac{\text{driving force}}{\text{resistance}}$$

The driving force is the temperature drop across the body, and the resistance is defined by Fourier's law:

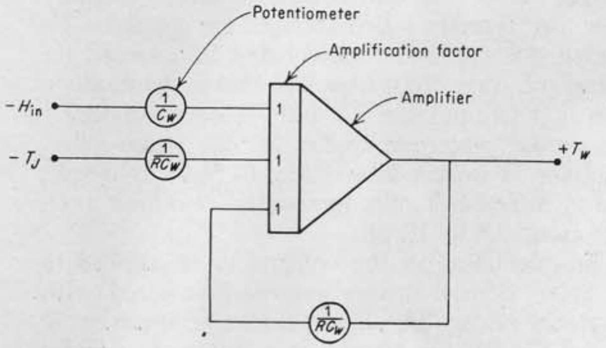
$$\frac{dQ}{dt} = \frac{\Delta T}{\gamma} \tag{1}$$

where

$\frac{dQ}{dt}$ = Rate of heat flow in Btu/min
ΔT = Temperature differential in deg F
γ = Thermal resistance in deg F/Btu/min

The similarity between Fourier's law and Ohm's

FIG. 3. Computer diagram for water-jacket section. The rectangle attached to the triangular amplifier symbol indicates that amplifier is connected as an integrator (feedback via series capacitance). Plus and minus signs indicate polarity of computer signal. Note that input signs are opposite to Equation 3A, because electronic integrator inverts signal.



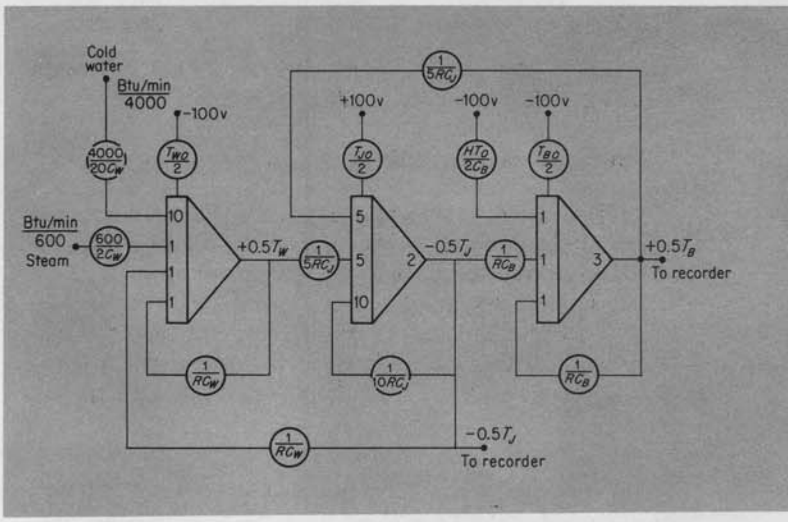


FIG. 4. Computer diagrams for entire jacketed kettle. Integrator outputs show scale factor of parameter as well as polarity.

law for electric current is immediately apparent, and an analog simulation is applicable. This similarity between heat flow and the flow of electricity (in a noninductive circuit) leads next to a definition of thermal capacity. Thermal capacity signifies the ability of a mass to store heat energy, and is determined by the specific heat of the substance. For example, the specific heat of water at standard conditions is unity; that is, if one pound of water is raised in temperature by 1 deg F, 1 Btu will be absorbed. Therefore, by definition, the units of thermal capacity for a specific mass are:

$$C_{thermal} = \text{Btu/deg}$$

The thermal capacities used in this analysis, C_w, C_j, and C_b, are the heat capacities of the water jacket, the jacket wall, and the batch product respectively. To simplify these relations, the assumption of one-dimensional heat flow was made.

To be theoretically correct, heat flow, like the flow of electric current, must be represented by a system having parameters distributed throughout the body. Mathematical treatment of distributed parameter systems involves partial differential equations. To simplify the analysis, the assumption of "lumped" parameters was made by assuming uniform temperature distribution throughout each section of the process.

For the simplifying assumption of uniform temperature in the water jacket, the following heat balance equation can be written:

$$\underbrace{C_w \frac{dT_w}{dt}}_{\substack{\text{Heat}\\\text{stored}}} = \underbrace{H_{in}}_{\substack{\text{Heat}\\\text{flow}\\\text{in}}} - \underbrace{\frac{T_w - T_j}{R}}_{\substack{\text{Heat}\\\text{flow}\\\text{out}}} \qquad (2)$$

This first-order differential equation is a simple statement of the physical fact that the rate of change of the water jacket temperature T_w is proportional to the difference between the flow of heat entering and leaving the jacket.

Transposing Equation 2 for the rate of change of the water jacket temperature gives:

$$\frac{dT_w}{dt} = \frac{H_{in}}{C_w} - \frac{T_w}{RC_w} + \frac{T_j}{RC_w} \qquad (3)$$

which can be set up directly on an analog computer (Figure 3) by integrating both sides*.

$$T_w = \int \frac{H_{in}}{C_w}\,dt - \int \frac{T_w}{RC_w}\,dt + \int \frac{T_j}{RC_w}\,dt$$

Using the same procedure, the heat-balance equation for the inner jacket wall can be written.

$$\underbrace{C_j \frac{dT_j}{dt}}_{\substack{\text{Heat}\\\text{stored}}} = \underbrace{\frac{T_w - T_j}{R}}_{\substack{\text{Heat}\\\text{flow}\\\text{in}}} - \underbrace{\frac{T_j - T_b}{R}}_{\substack{\text{Heat}\\\text{flow}\\\text{out}}} \qquad (4)$$

For the product batch

$$\underbrace{C_b \frac{dT_b}{dt}}_{\substack{\text{Heat}\\\text{stored}}} = \underbrace{\frac{T_j - T_b}{R}}_{\substack{\text{Heat}\\\text{flow}\\\text{in}}} - \underbrace{h(T_b - T_a)}_{\substack{\text{Heat}\\\text{flow}\\\text{out}}} \qquad (5)$$

These equations can be set up in exactly the same way as the one for the water jacket, but with different coefficient potentiometer values. These circuits, when connected as shown in Figure 4, yield the complete computer diagram for the process.

Since the control valves are arranged to admit both steam and cooling water to the water jacket, the heat input into the simulated process is made up of two components (shown in Figure 4). The coefficient potentiometers labeled $T_{wo}/2$, $T_{jo}/2$, and $T_{bo}/2$ establish initial process temperature at the start of the computation. The signal shown at the output of

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* *Electronic Analog Computers*, Granino A. Korn and Theresa M. Korn, McGraw-Hill Book Co., Inc., New York, 1952.

the amplifier identifies the process variable at that point with its polarity and its scale factor.

Control valve simulation

The process temperature T_b is controlled by modulating the flow of steam or cooling water into the water jacket according to the output of an industrial pneumatic controller. This entrance of steam or cold water displaces equivalent quantities of water at temperature T_j. The control valves used to control these flows are diaphragm-operated and pneumatic, and are arranged to perform as in Figure 5.

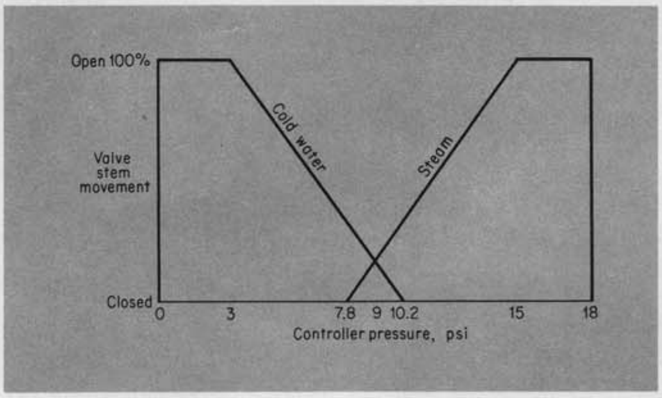


FIG. 5. Action of control valves vs. controller output pressure.

For controller output pressures from 3 psi to 10.2 psi, the cooling-water valve moves from full open to completely closed, while for pressures of 7.8 psi to 15 psi, the steam valve moves from full closed to full open. Thus, the response of each valve overlaps in the region of 9 psi.

The flow through a valve at a given opening is a simple function of the maximum flow (W_w) that can be delivered with the same pressure drop. This function can be simulated on an analog computer by using a servo multiplier, which is a servo-driven series of potentiometers. The controller pressure, represented as a machine voltage, drives a servo amplifier and motor to position the series of potentiometers according to a signal fed back from a reference voltage on one potentiometer. The voltage representing maximum flow is applied across another potentiometer and the actual valve flow is represented by the voltage fraction picked off by the arm position. To simulate equal percentage characteristics, the arm of the potentiometer is loaded with a resistor whose value is approximately one-tenth of the potentiometer resistance.

The computer circuit used to simulate the cold water and steam valve is shown in Figures 6A and B. In this case, P_c has been chosen to have an excursion of 75 volts, equivalent to 0—15 psi. From this input is subtracted the 7.8-psi voltage equivalent for the steam valve and the 3-psi voltage equivalent for the cold water valve, so no valve operation occurs when P_c is below these values. The diodes on the output of amplifier 31 limit its negative voltage output. This effectively "cuts off" the action of the cold water valve for P_c greater than 10.2 psi (Figure 5). Servo-driven potentiometers 2B and 2A will give the water flow and the heat flow respectively when the correctly scaled voltages are applied. To simulate the cold-water valve being fully closed for control pressures of 10.2 psi and above, the negative sides of potentiometers 2A and 2B are grounded.

A similar procedure applies for the steam valve. Amplifier 32 is biased by diodes to limit its output to 0 to minus 100 volts for controller output pressures from 7.8 to 15 psi.

The calculation of the voltages to be applied to the valve potentiometers can be illustrated with the steam valve. The heat content of steam at 50 psi is 1,174 Btu $= H_s$ and at a maximum flow of 10 lb per min for the valve wide open, gives a heat flow of 11,740 Btu min. This steam condenses and assumes the jacket-water temperature, the heat content of which is $10(T_w—32)$. Hence, heat liberated by steam to the jacket water is:

$$11{,}740 - 10\,(T_w - 32) = 12{,}060 - 10.0\,T_w$$
$$\text{or } W_s H_s - W_s\,(T_w - 32) = (W_s H_s + W_s\,32) - W_s T_w$$

Figure 6A shows the computer connections required to simulate the heat flow through the valve. The output voltage at the arm of potentiometer 4A constitutes the heat flow into the process (Figure 4) from the steam valve, divided by the scale factor of 1 volt = 600 Btu min.

A similar procedure is involved in scaling the heat carried away by the cold water. For this case the scale factor is 1 volt = 4,000 Btu min. This output is introduced to the process of Figure 4 through an attenuator.

Simulation of temperature transmitter

Tests have shown that the dynamic characteristics of the resistance bulb, instrument, and transmission can be approximated by two first-order lags with 4-sec time constants. The transfer function is:

$$\frac{P_c}{T_b}(s) = \frac{1}{(4s+1)\,(4s+1)}$$

The performance of the temperature transmitter as described by this equation is easily simulated on the analog computer.

One of the advantages of analog computation is the ability to transform the time scale of a particular problem. In this process, transients up to 30 min are involved and a time-scale change of 60 to 1 has been made.

With the time scale change factor $B=1/60$, this transmitter transfer function becomes:

$$\frac{P_c}{T_b}(s) = \frac{1}{(4B_s+1)\,(4B_s+1)}$$

The computer circuit which is employed to simu-

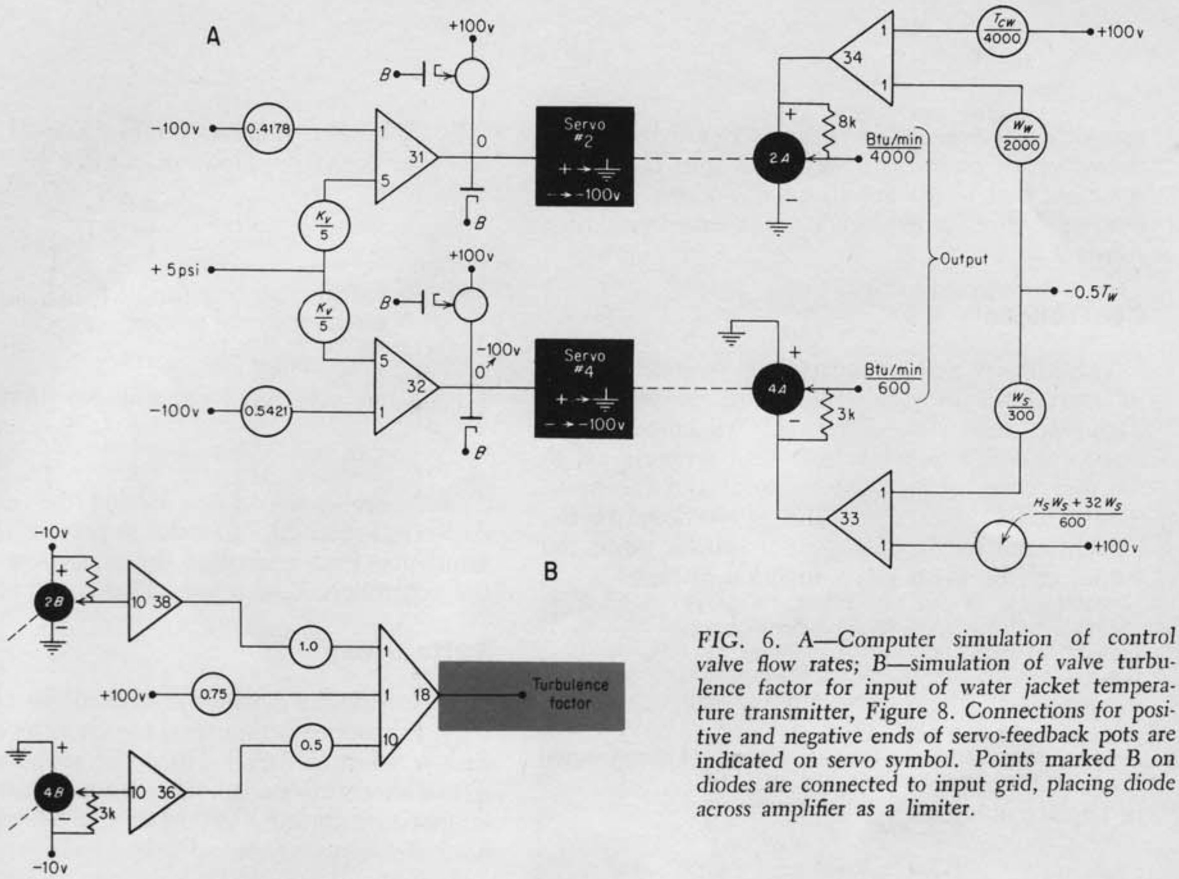


FIG. 6. A—Computer simulation of control valve flow rates; B—simulation of valve turbulence factor for input of water jacket temperature transmitter, Figure 8. Connections for positive and negative ends of servo-feedback pots are indicated on servo symbol. Points marked B on diodes are connected to input grid, placing diode across amplifier as a limiter.

late the temperature transmitter is shown in Figure 7.

The batch-temperature-transmitter output of 3-15 psi is the equivalent of 15-75 volts in the computer. The first amplifier is biased so that its output range is plus 42.5 to minus 40 volts for the entire range of T_b. The diode rectifier limits its output to 0 to minus 40 volts. Amplifiers 4 and 5 are connected as first-order lags with time constants of $B\tau = 1/15$-sec computer time. The last summing amplifier multiplies the 40 volts by 1.5 and adds a constant 15 volts (3 psi) to give the equivalent output pressure in volts.

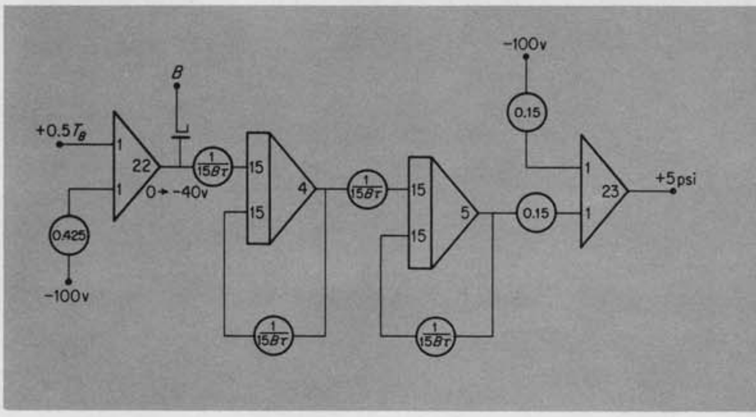
Jacket-water temperature

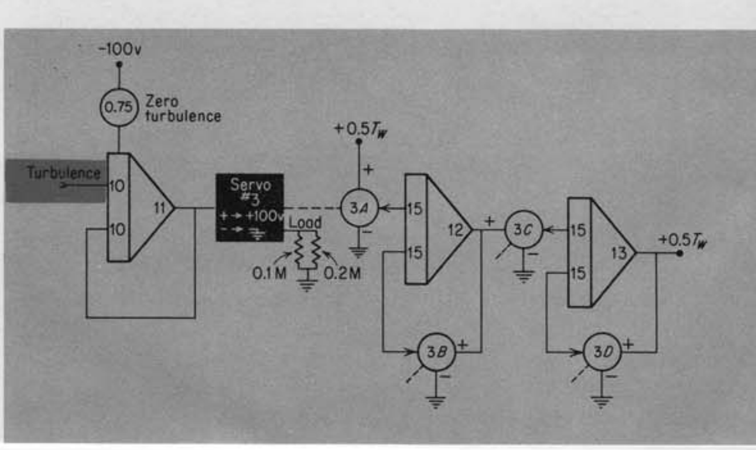
The jacket-water temperature transmitter transmits a 3-to-15-psi pneumatic pressure proportional to the water temperature. The range of this instrument is 0-250 deg F. Its dynamic characteristics can be represented by two first-order lags. However, the time constant varies with the amount of water turbulence, being approximately 4 sec for high turbulence and 6 sec with no turbulence.

The computer circuit used to simulate these conditions is similar to the circuit for the batch-temperature transmitter and is shown in Figure 8. In this case, however, instead of being set at a fixed value, the potentiometers are made to vary automatically as a function of turbulence. This is done by using the servo-driven potentiometers of a servo multiplier whose input is a function of a selected

FIG. 7. Computer circuit for batch temperature transmitter. The τ indicates that the computer is not operating in real time—i.e., that time has been scaled. In this case time has been speeded up by a factor of 60.

FIG. 8. Circuit for jacket-water temperature transmitter. Temperature input is via a scaling pot varied by valve turbulence. Turbulence servo-feedback pot arm is loaded to proper nonlinearity.





turbulence factor. This factor is found by adding the effects of steam and water flow into the jacket, assuming that 20-percent steam flow alone produces maximum turbulence (giving a thermo-well time constant of 4 sec).

Controllers

The primary and secondary loop controllers used to control the process temperature are pneumatic stack-type industrial controllers. Two-mode pneumatic controllers provide an output proportional to the difference between the set-point and the measured variable (the error), and proportional to the time integral of the error. In equation form, the output can be written as a function of time:

$$P_o(t) = K_o\epsilon + \frac{K_o}{\tau_c}\int \epsilon dt \qquad (7)$$

Where

P_o = controller output
K_o = controller gain
ϵ = error difference between desired and actual output
τ_c = reset time constant

In Laplace notation:

$$P_o(s) = K_c\epsilon + \frac{K_c}{\tau_c s}\epsilon \qquad (8)$$

The computer circuit is easily obtained by summing the two terms as shown in Figure 9.

FIG. 9. Circuit for simulating theoretical two-mode controller.

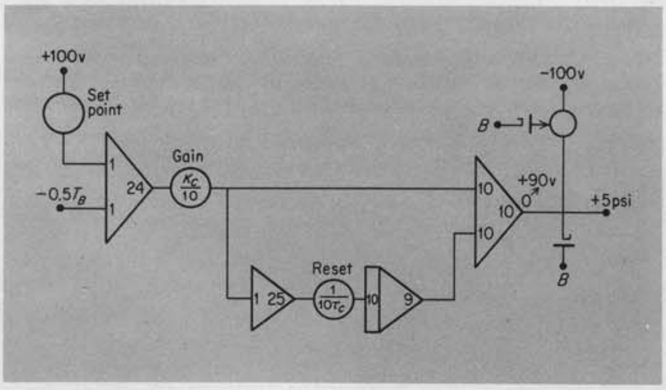
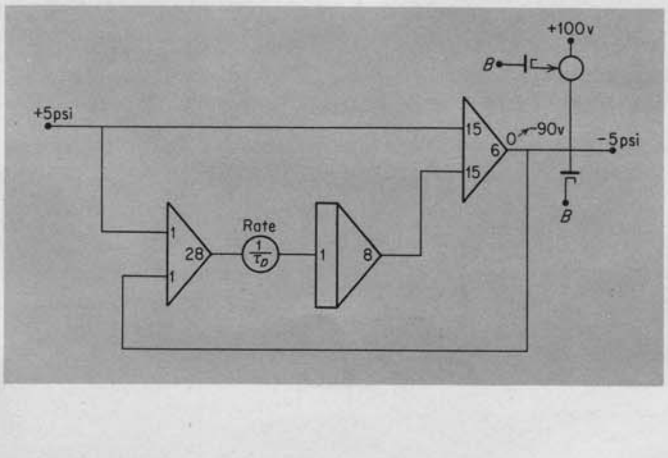


FIG. 10. Circuit for compensated derivative controller.



By definition, the transfer function of the compensated derivative mode of control is:

$$\frac{P_o}{P_i}(s) = \frac{\tau_d s + 1}{\alpha \tau_d s + 1} \qquad (9)$$

Where

P_o = output pressure of derivative controller
P_i = measured process variable
τ_d = derivative time constant
α = compensating gain — 1/15

Solving this equation for the output pressure gives:

$$P_o(s) = \frac{P_i}{\alpha} + \frac{1}{s} \times \left[\frac{P_i - P_o}{\alpha \tau_d}\right]$$

The computer circuit for solving this equation is shown in Figure 10. In order to prevent the voltage simulation from exceeding the maximum output of the controllers, diodes are added.

Ratio relay

Neglecting the dynamic characteristics of the ratio relay, this device is simulated merely as an attenuator with a value of 0.65. Thus, the set-point for the secondary controller is held at a maximum desirable temperature during start-up and maximum output from the primary controller.

Computer operation

The computation procedure consists of combining all computer diagrams into a complete circuit by connecting related variables. From this diagram the computer pre-patched panel is prepared by making patchcord connections between the various amplifiers, multipliers, and attenuators. The patch panel is then inserted into the computer console patch-bay, and the reference voltage is turned on. The values of the various coefficient potentiometers are then set with the aid of the digital voltmeter. By placing the programmed patch panel in the patch-bay before the potentiometers are adjusted, any possible error due to imperfect loading can be avoided.

The first step in the computation usually is to check the operation of the various components in the system. For instance, a known flow rate of steam is introduced into the circuit and the resultant process temperature is monitored to check this part of the circuit. The circuits for the valves and controllers are also isolated and checked both dynamically and statically for ranges and accuracy. Once all components have been checked individually, the system can be reconnected and the computation can begin.

It is best to establish the validity of the simulation by making a series of check solutions. A check solution establishes the fact that the simulation operates similarly to the existing system or else agrees closely with steady-state calculations. Once this is established, the investigation of the system-design changes for satisfactory performance can be made with complete confidence.

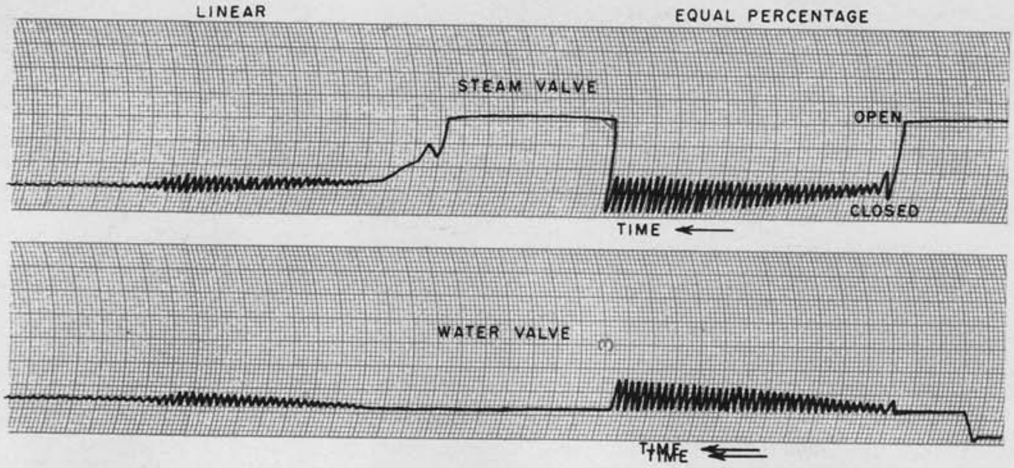
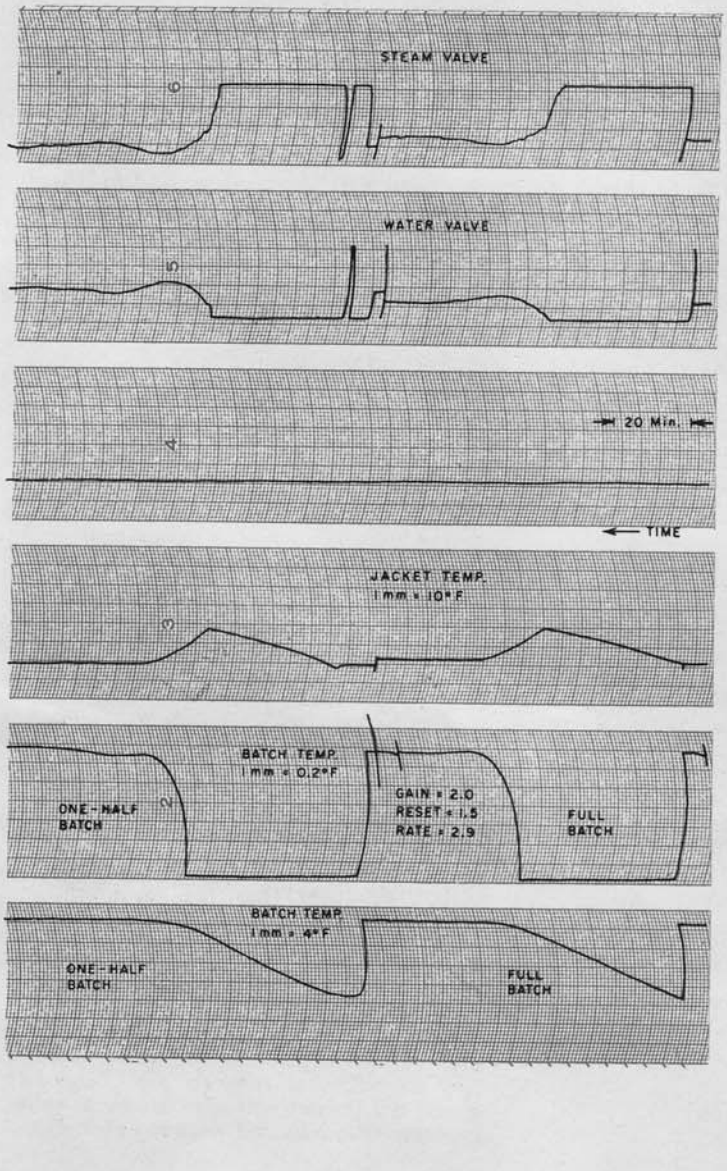


FIG. 11. Instability due to nonoptimum controller settings at first indicated linear valve trim was better than equal percentage trim; this was later disproved. Note that in all recorder charts time increases from right to left.

FIG. 12. Effect of change in batch size from full to one-half. T_b remained within 0.4 deg F.



THE RESULTS

During this simulation several runs were made, varying many of the parameters. The output of the computer was recorded and a record kept of each change. Changes which would have taken months in the field were completed in seconds with the computer. In addition, data were obtained in a form which was readily understood without any compromise in conditions such as can occur in field tests.

The results of the computation are shown in Figures 11 to 16. Note particularly that the curves advance from right to left with increasing time. Channels 5 and 6 show the movement of the valves. Channels 1 and 2 show the batch temperature—that in channel 2 being expanded in scale. Channel 3 shows the jacket temperature T_w.

By using the computer for this problem, at least one man-month of an engineer's time was saved. In addition, much equipment that otherwise would have been purchased was not required. The best control system for the process was derived and a better knowledge of the influence of varying parameters and the areas of critical adjustments was obtained.

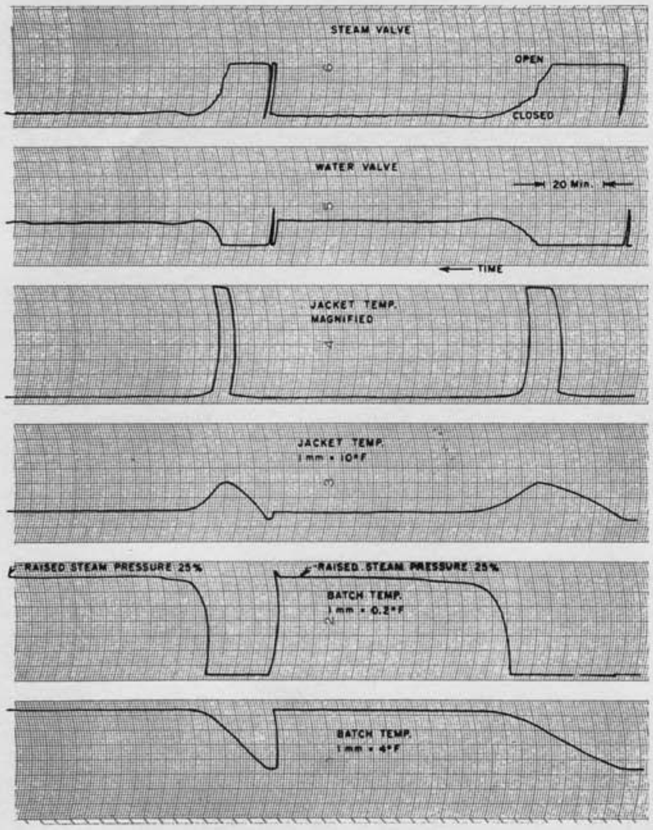


FIG. 13. Sudden 25-percent increase in steam pressure had no effect on cascade temperature control. Single-loop system showed a disturbance of more than 1 deg F.

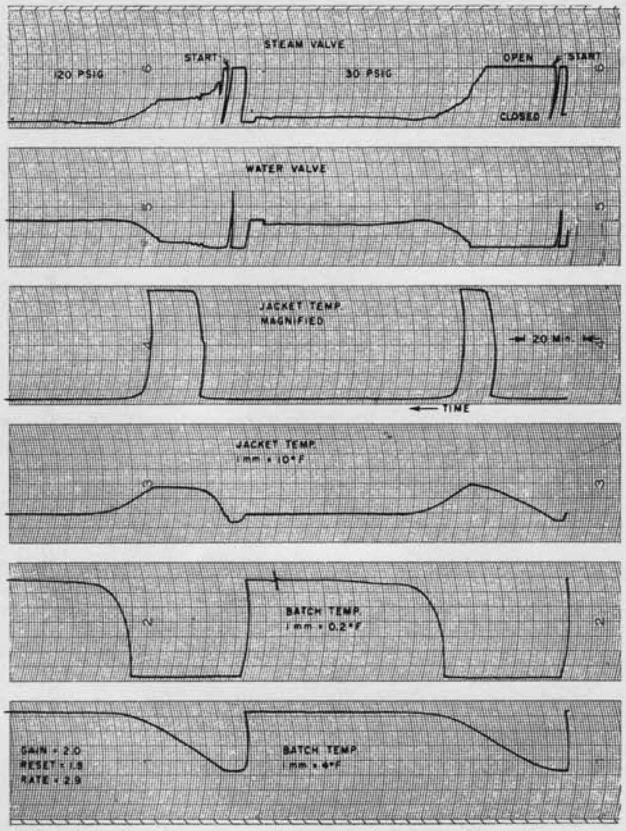


FIG. 14. A 400-percent increase in steam flow also shows favorable results. Single-loop system showed 5-deg-F overshoot and considerable valve cycling.

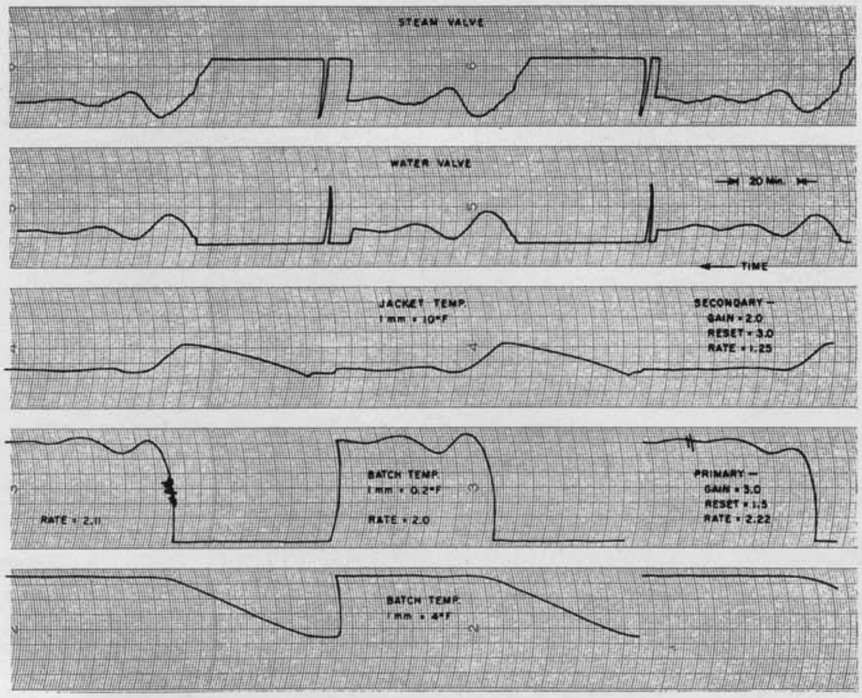


FIG. 15. Rate adjustment on primary controller proved critical. Existing controllers do not provide the precision of rate adjustment needed for this control application. Controller rate action would have to be stabilized against drift.

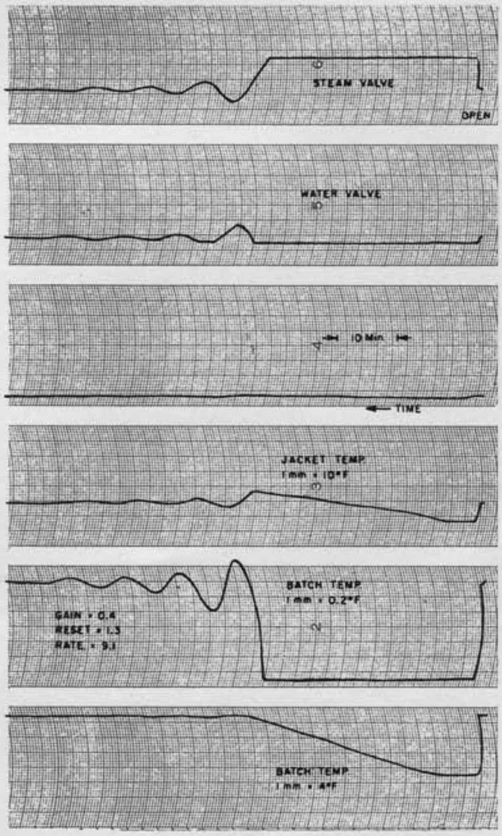


FIG. 16. Best response for any setting of single-loop controller was relatively poor. Single-loop system tied primary controller directly to valves, bypassing ratio relay and secondary controller.

Home Office

ELECTRONIC ASSOCIATES, INC.
Long Branch New Jersey
Telephone: CApital 9-1100

Central Regional Office

ELECTRONIC ASSOCIATES, INC.
101 South Pine Street Mount Prospect, Illinois
Telephone: CLearbrook 5-6070

Western Regional Office

ELECTRONIC ASSOCIATES, INC.
5437 Laurel Canyon Blvd., Suite 212, North Hollywood, Calif.
Telephone: POplar 3-7371

European Regional Office

ELECTRONIC ASSOCIATES, INC.
43 Rue de la Science Brussels, Belgium
Telephone: Brussels 11-43-69

Princeton Computation Center

ELECTRONIC ASSOCIATES, INC.
P. O. Box 582 Princeton, New Jersey
Telephone: PRinceton 1-2291

EAI COMPUTATION CENTER AT LOS ANGELES, Inc.
1500 East Imperial Highway El Segundo, California
Telephone: EAstgate 2-3220

European Computation Center

ELECTRONIC ASSOCIATES, INC.
43 Rue de la Science Brussels, Belgium
Telephone: Brussels 11-43-69

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