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INTRODUCTION TO ELECTRONIC ANALOGUE COMPUTING

by

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## PREFACE

This experiment manual has been prepared to introduce participants in the International Institute of Nuclear Science and Engineering to the use of analog computers. The problems chosen for demonstration are intended as general examples of analog computer techniques, but their solutions are also interesting to students of reactor engineering. Each experiment has been worked out in sufficient detail to permit students to utilize most of their class time becoming familiar with the operation of a specific analog computer.

The manual consists of four sections:

1. General theory of electronic analog computers.
2. Operational information for the Electronics Associates PACE computer.
3. Programming an analog computer.
4. Experiments.

## I. THEORY OF ELECTRONIC ANALOG COMPUTERS

### A. Introduction to Analog Computation

The oldest example of analog computation is the geometrical scaling used in mechanical drawing. With the introduction of logarithms, the first analog computing machine for the generalized mathematical operation of multiplication came into being. Concurrent with the development of these crude forms of analog computers was the development of the abacus digital-type computer.

An early problem in the solution of ordinary differential equations was to find a mechanical system whose time dependence was equivalent or analogous to a given electrical circuit. This procedure requires considerable imagination and understanding of different kinds of physical systems and, while quite economical for solving a particular problem, lacks versatility. Rather than solve a differential equation by seeing the analogy physically, it is possible to devise a computer which performs the operations of integration, addition and multiplication which are required mathematically to solve a differential equation. This type of computer is properly called a differential analyzer. The original differential analyzer performed the required operations by means of mechanical linkages.

Beginning about 1940 the operations required of a differential analyzer were successfully accomplished with electronic equipment. These electronic differential analyzers are now generally referred to as analog computers.

Analog computers are most often used for solving ordinary differential equations, though they can be used for algebraic equations and partial differential equations as well. Analog computers can also be used as curve-fitting devices.

Present-day analog computers are available with an accuracy of 0.01%. Digital machines can be programmed to give much more accuracy, but analog machines continue to dominate in certain applications. These two types of computers differ in many respects, the most important difference being that analog machines can actually perform the continuous operations of differentiation and integration, whereas a digital machine (which can only count) must resort to finite-difference approximations.

In reactor engineering the following problems are frequently solved by means of the analog computer: (1) automatic control rod drives; (2) stability of reactor power plants with internal feedbacks; (3) studies of reactivity lifetime of reactor fuels; (4) dynamics of heat transfer and

coolant flow. An extensive list of analog computer applications to other engineering fields can be found in *Electronic Analog Computers* by Korn and Korn.<sup>(1)</sup>

## B. The Electronic Analog Computer

An analog computer is a computer in which problem variables are represented by physical quantities, such as voltages, rotations or displacements. The PACE computer is a voltage analog, so that problem variables are represented by voltages. Since this requires much electronic equipment, the PACE computer is called an electronic analog computer.

In solving a problem with the electronic analog computer, the dependent variable of the problem is represented by a voltage in the computer. The voltage range of the computer variable is frequently limited to the range from -100 to +100 volts. If the independent variable does not appear explicitly in the problem, the computer represents the independent variable by time. If the independent variable of the problem is time and the proportionality between the problem time scale and the computer time scale is unity, the computer solves a problem in real time and is said to be functioning as a simulator.

In problems where the independent variable appears explicitly, it is necessary to represent this variable as a voltage in the computer as well as by time.

The machine variable voltages of the analog computer must now be manipulated by means of circuits to achieve mathematical operations of addition, subtraction, multiplication, division, integration and differentiation. In addition, it is frequently desirable to generate arbitrary functions which do not have simple analytical expressions.

The analog computer is an assembly of specific circuits which perform basic mathematical operations. The number of operations that the computer can perform is limited by the degree to which the circuits are duplicated. These computing components must possess a control system which sets them into operation simultaneously at the beginning of the solution of the problem.

The solution of the problem is, of course, a machine variable voltage and as such can be used to actuate an electromechanical recorder. Special purpose recorders are available for analog computers, which record solutions of problems on standard graph paper.

In order to understand and use the electronic analog computer, one must understand the theory of the component parts, methods of interconnecting these parts and methods of control (adjustment and start-stop procedure).



## C. Electronic Analog Computer Components

### 1. High-gain dc (direct-coupled) Amplifier

The basic component of the electronic analog computer is the high-gain direct-coupled amplifier. The high-gain amplifier is represented by the symbol shown in Figure 1, where  $e_i$  and  $e_o$  are related to a ground or zero potential, which is not shown.

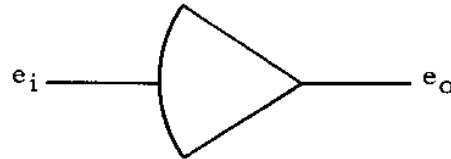


Figure 1  
High-gain dc amplifier

The high-gain amplifier is an electronic amplifier with an odd number of stages so that the output voltage  $e_o$  is opposite in polarity to the input voltage  $e_i$ . The amplifiers are stabilized so that when  $e_i = 0$ , then  $e_o = 0$  also.

If  $A$  represents the gain of the amplifier,  $e_o = -Ae_i$  (where  $A$  is approximately  $10^8$ ).

### 2. Operational Amplifier

If input and feedback networks are connected to a high-gain dc amplifier, it becomes an operational amplifier. By suitable choice of impedances in the networks, the amplifier may be made to perform the operations of

- a. addition and subtraction;
- b. multiplication by a constant;
- c. integration;
- d. differentiation (not recommended since it is a noise-amplifying process); or
- e. combinations of the above.

Figure 2 is a diagram of an operational amplifier.

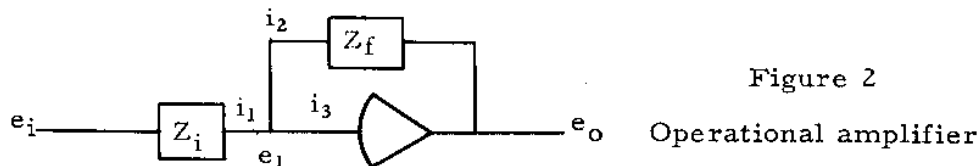


Figure 2

Operational amplifier

It is necessary to find a relation among  $e_i$ ,  $e_o$ ,  $Z_i$  and  $Z_f$ . A circuit description follows.

Since the input grid of the high-gain amplifier has a very high impedance, the current  $i_3$  is very small and can be neglected.

Therefore

$$i_1 = i_2$$

$$i_1 = (e_i - e_1)/Z_i$$

$$i_2 = (e_1 - e_o)/Z_f$$

$$e_o = -Ae_1$$

Equating the expressions for  $i_1$  and  $i_2$ , there is obtained

$$(e_i - e_1)/Z_i = (e_1 - e_o)/Z_f$$

Substituting  $e_1 = -e_o/A$  into the above

$$\frac{e_i + (e_o/A)}{Z_i} = -\frac{(e_o/A) + e_o}{Z_f}$$

Solve for  $e_o/e_i$ :

$$\frac{e_o}{e_i} = -\frac{Z_f}{Z_i} \left[ 1 + \frac{1}{A \left( \frac{Z_f}{Z_i} + 1 \right)} \right]$$

If  $A$  is sufficiently large,

$$\boxed{e_o/e_i = -Z_f/Z_i}$$

If  $Z_i = Z_f = 1 \text{ M}\Omega$ , then (see Figure 3)

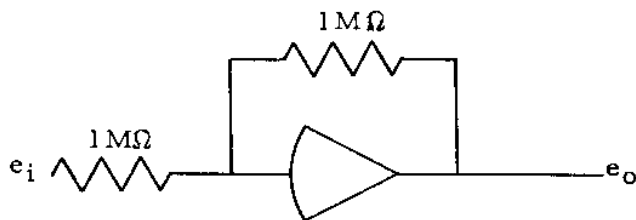


Figure 3

Operational amplifier with resistive input and feedback networks

$$e_o = -e_i.$$

If  $Z_i$  is  $< Z_f$ , then  $e_o = -Ke_i$ , where  $K > 1$ . In general  $Z_f$  is  $1\text{ M}\Omega$  if it is a resistor and  $Z_i$  is  $1\text{ M}\Omega$ ,  $0.1\text{ M}\Omega$  or  $0.2\text{ M}\Omega$ , giving gains of 1, 10 and 5, respectively.

The symbol for this operational amplifier is shown in Figure 4,

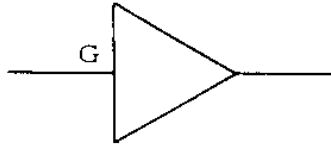


Figure 4

Symbol for an operational amplifier that multiplies by a constant

where  $G$  represents the gain of the amplifier determined by  $Z_f/Z_i$ . To make an operational amplifier perform addition, parallel input impedances are used.

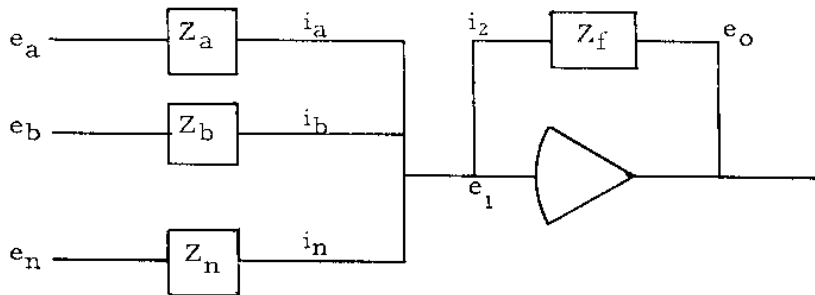


Figure 5

Operational amplifier with parallel input impedances

In Figure 5,

$$i_a + i_b \dots + i_n = i_2 .$$

But

$$e_1 = -e_o/A \cong 0 .$$

Therefore

$$i_k = e_k/Z_k$$

$$\frac{e_a}{Z_a} + \frac{e_b}{Z_b} + \dots + \frac{e_n}{Z_n} = -\frac{e_0}{Z_f} .$$

Therefore

$$e_0 = -Z_f \sum_{i=1}^n \frac{e_i}{Z_i} .$$

If  $Z_f$  and each  $Z_i$  are resistors, then addition of  $e_1, e_2 \dots e_n$  is performed. The symbol shown in Figure 6 is used for the summing operational amplifier.

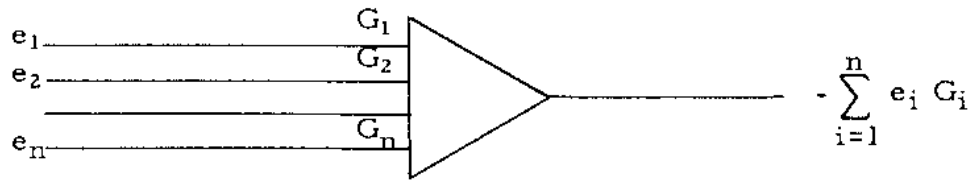


Figure 6

Symbol for summing operational amplifier

$$G_i = \frac{Z_f}{Z_i} = \frac{R_f}{R_i} \text{ represents the gain given to } e_i .$$

Integration is accomplished by the circuit shown in Figure 7.

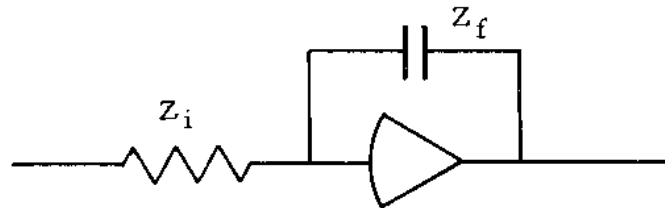


Figure 7

Integrator

If

$$Z_i = R \quad (R \text{ in ohms})$$

and

$$Z_f = \frac{1}{j\omega C} \quad , \quad (C \text{ in farads})$$

$$\frac{e_0}{e_i} = -\frac{1}{j\omega RC}$$

But

$j\omega = p = \frac{d}{dt}$  for functions that can be expressed in terms of  $\sin \omega t$  and  $\cos \omega t$ . For if

$$y = \sin \omega t$$

then

$$\frac{dy}{dt} = py = \omega \cos \omega t$$

$$\frac{d^2y}{dt^2} = p^2y = -\omega^2 \sin \omega t$$

Thus

$$p^2 = \frac{d^2}{dt^2} = -\omega^2$$

and

$$p = \frac{d}{dt} = j\omega$$

Therefore,

$$\frac{e_0}{e_i} = -\frac{1}{pRC}$$

Since

$$\frac{1}{p} = \int dt$$

it follows that

$$e_0 = -\frac{1}{RC} \int e_i dt$$

In Figure 8 is the symbol for a single input integrator, where  $G = 1/RC$ .



Figure 8  
Symbol for integrator

In Figure 9 is the symbol for a multiple input integrator.

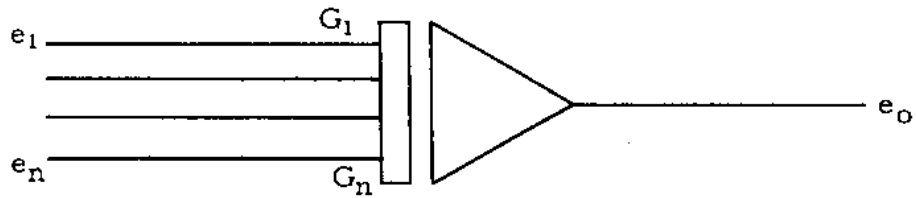


Figure 9  
Multiple input integrator

Here

$$e_0 = -\frac{1}{C} \int \sum_{i=1}^n \frac{e_i}{R_i} dt = - \int \sum_{i=1}^n G_i e_i dt \quad ,$$

where

$$G_i = 1/R_i C \quad .$$

Figure 10 shows the diagram for a single input differentiator.

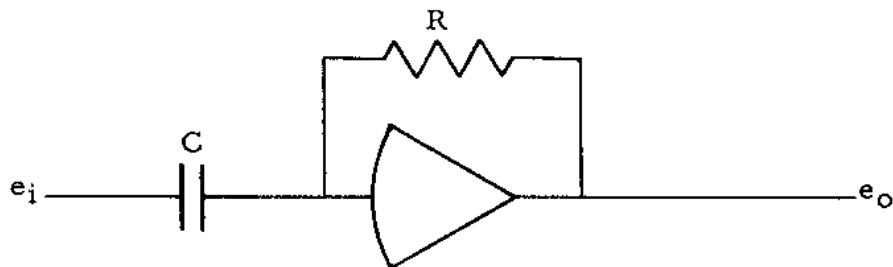


Figure 10  
Differentiator

In this case

$$\frac{e_0}{e_i} = - \frac{R}{\frac{1}{j\omega C}} = - pRC$$

and

$$e_0 = -RC \frac{de_i}{dt} \quad .$$

The use of differentiating circuits is to be discouraged because the output is a derivative of the true input signal as well as the derivative of any noise present. Since the rate of change of the noise is more rapid than that of the input signal, the noise is amplified and the output distorted.

Other differentiating circuits are presented in books by Johnson<sup>(4)</sup> and by Chestnut and Mayer.<sup>(2)</sup>

An alternative method of solving the response of an operational amplifier is to regard it as an inverse feedback system.

The actual schematic diagram of the summing amplifier is shown in Figure 11.

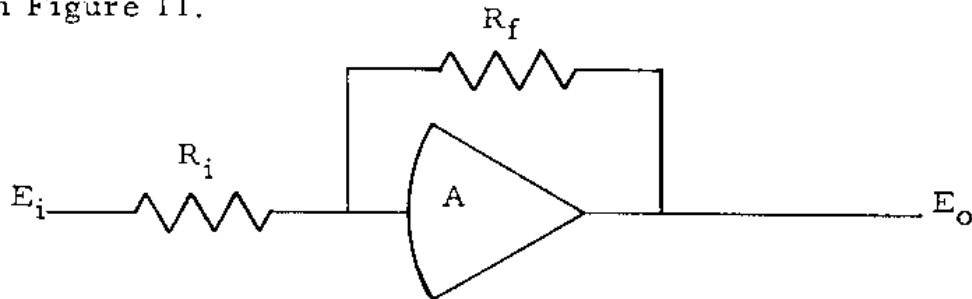


Figure 11

Schematic diagram of summing amplifier

This can be redrawn as the feedback amplifier shown in Figure 12, for

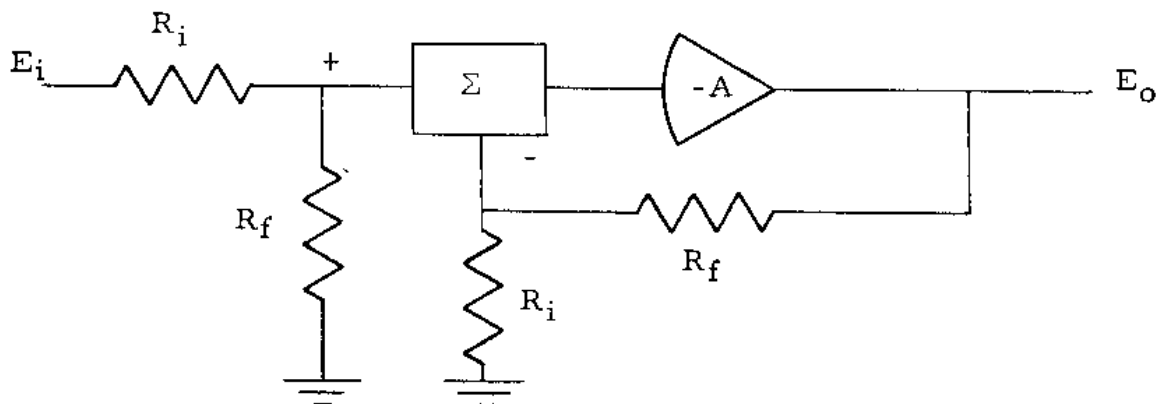


Figure 12

Feedback diagram for a summing amplifier

$$\frac{E_o}{E_i} = + \frac{R_f}{R_i + R_f} \left[ - \frac{A}{1 + A \left( \frac{R_i}{R_f + R_i} \right)} \right] = - \frac{R_f}{R_i + R_f} \frac{(R_f + R_i) A}{(R_f + R_i + AR_i)} \approx - \frac{R_f}{R_i}$$

The same sort of feedback picture (see Figure 13) can be taken for the integrator, for

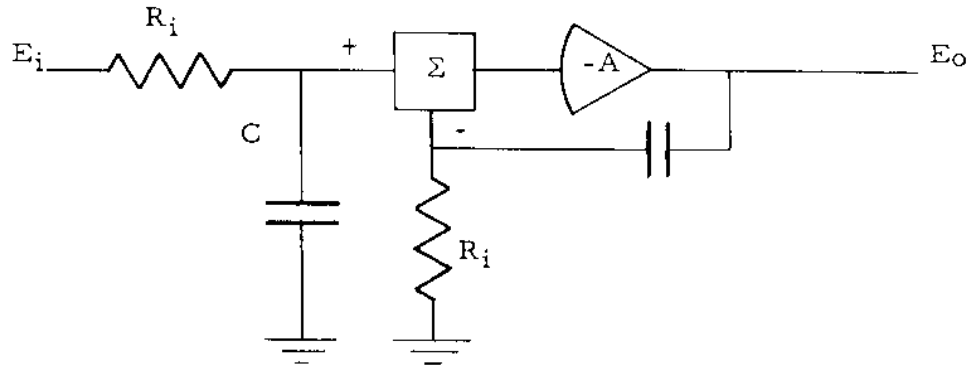


Figure 13

Feedback diagram for an integrator

which

$$\frac{E_o}{E_i} = + \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R_i} \left[ - \frac{A}{1 + \frac{1}{\frac{1}{j\omega C} + R_i}} \right] = \frac{1}{j\omega R_i C + 1} \left[ \frac{(j\omega C R_i + 1) A}{1 + j\omega R_i C + \frac{AR_i}{Cj\omega}} \right]$$

$$\approx - \frac{1}{j\omega R_i C} = - \frac{1}{p} \frac{1}{R_i C}$$

The diagram of a high-gain operational amplifier is shown in Figure 14.

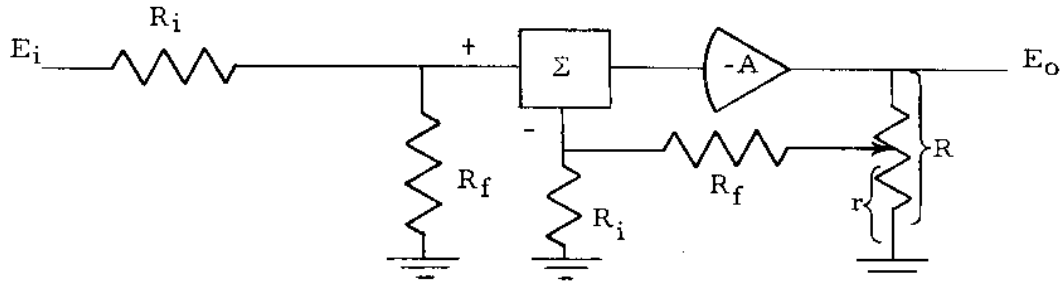


Figure 14

Feedback diagram for a high gain operational amplifier



(R is the resistance with respect to ground with  $R_f + R_i$  connected as shown.)

In this case,

$$\frac{E_0}{E_i} = + \frac{R_f}{R_i + R_f} \left[ - \frac{A}{1 + A \frac{r}{R} \frac{R_i}{R_f + R_i}} \right] \cong - \frac{R_f}{R_i} \frac{R}{r}$$

Since

$$r < R \text{ and } E_0 > E_i \quad .$$

### 3. Potentiometers

In Figure 15 are shown the electrical and analog computer symbols for a potentiometer.

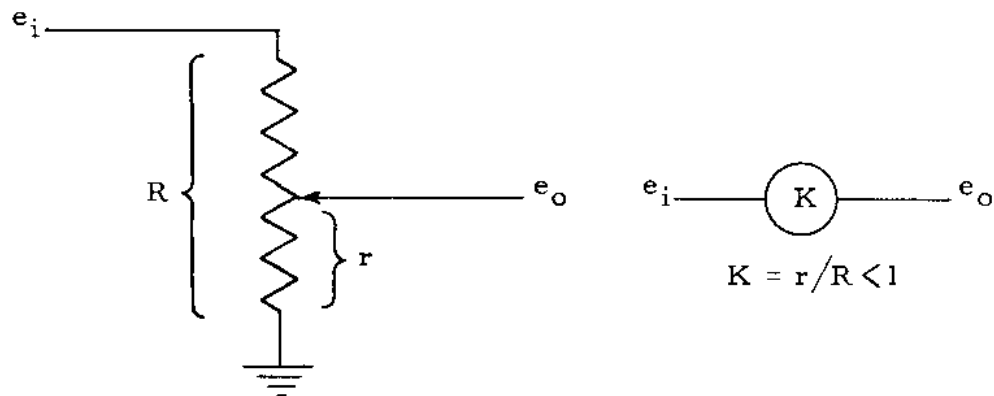


Figure 15

Potentiometer symbols

When a potentiometer is used in conjunction with a summing amplifier, the input variable  $e_i$  can be multiplied by the constant  $M = KG$  (see Figure 16).

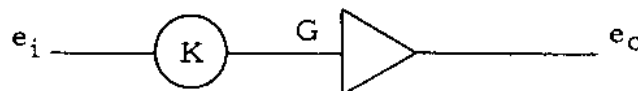


Figure 16

Multiplying by a constant

$$e_0 = - M e_i = - K G e_i \quad .$$

Note that  $0 \leq M < G$ .

Potentiometers are usually set by applying a fixed  $e_i$  of 100 volts to the input and adjusting the arm so that  $e_o = 100K$  volts. This must be read with a high-impedance voltmeter. Then for any input,  $e_o = Ke_i$ .

#### 4. Servomultipliers

Since a potentiometer permits multiplication of  $e_i(t)$  by  $r/R$ , it is observed that if  $r$  can be made to vary proportionally to a second voltage  $e_2(t)$ , the output would be a product of  $e_i(t)$  and  $e_2(t)$ . To accomplish the variation in  $r$  proportional to  $e_2(t)$ , the voltage  $e_2(t)$  is used to drive a position servomechanism which positions the arm of a potentiometer.

The servomechanism drives six identical potentiometers, the first of which is called the follow-up pot (F.U.), which has two ends and a center tap. One end is marked  $+$  and is in the direction that the servomotor will rotate when an input  $e_2$  goes more positive. The other end is marked  $-$  and is in the direction that the servomotor will rotate when the input  $e_2$  goes more negative (see Figure 17).

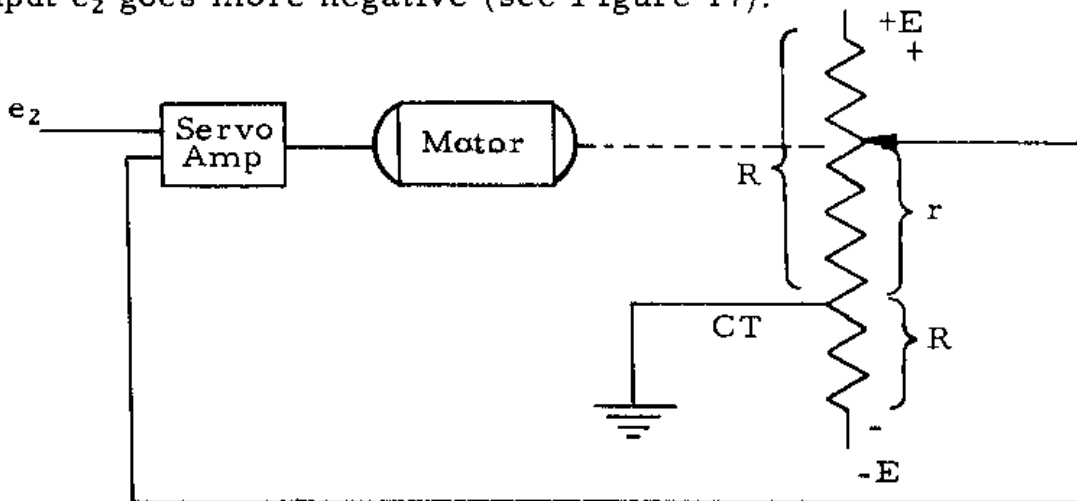


Figure 17

Servomultiplier input circuit

If  $e_2 = +E$ , then the arm will be at the  $+$  end of the pot. If  $e_2 = KE$ , then the arm will be driven a distance  $K = r/R$  toward the  $+$  end.

As previously mentioned, the wipers of six identical potentiometers are ganged together. This makes it possible to multiply one voltage by five others.

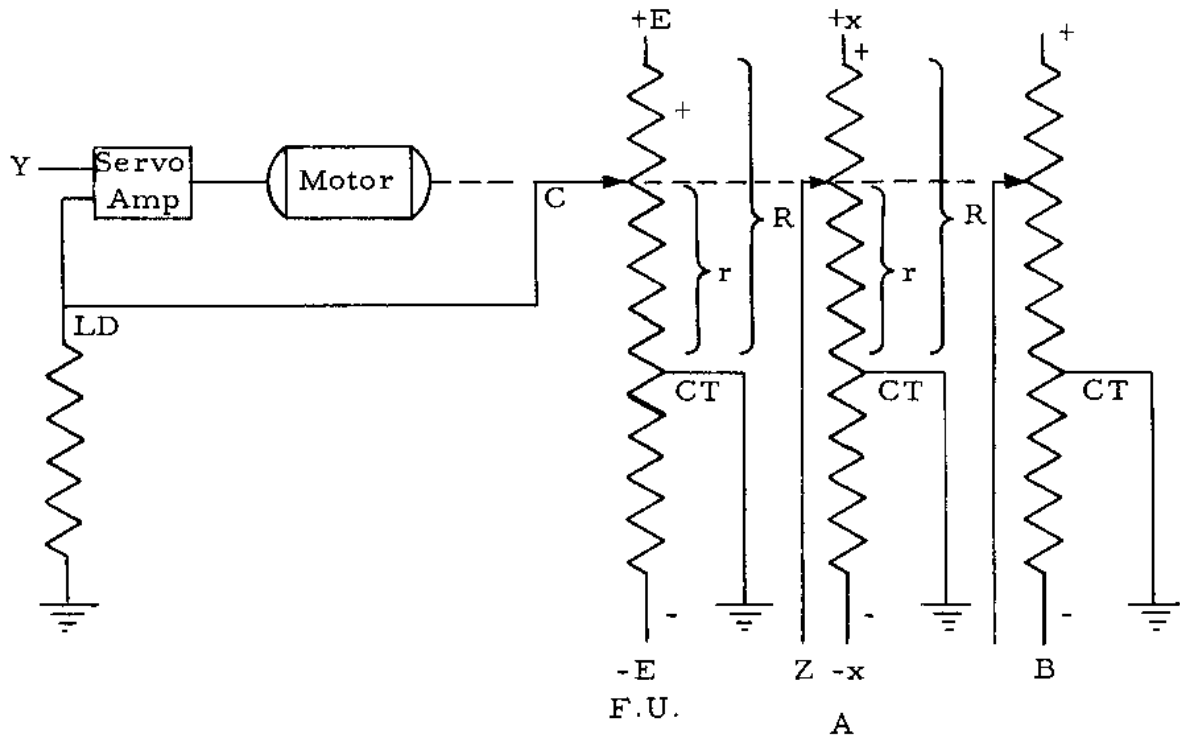


Figure 18

Servomultiplier circuit

If (see Figure 18)  $Y > 0$ , then the servomotor drives the arm until  $C = + (r/R)E = Y$ . This positions all wiper arms in the same manner. Since  $+X$  is applied to the plus side of A and  $-X$  is applied to the - side of A,

$$Z = (r/R)X \quad .$$

But

$$(r/R)E = Y$$

so that

$$r/R = Y/E \quad .$$

Then

$$Z = XY/E \quad .$$

If  $-Y$  drives the servomultiplier,

$$Z = -XY/E \quad .$$

It is important to notice that no sign reversal takes place with this circuit. Generally,  $E$  is taken to be 100 volts.

The input to the servomultiplier is constructed so that it represents an infinite impedance when it is balanced. Therefore, without LD (load resistor), the F.U. pot (or cup) is not loaded. The other 5 cups will have their wipers connected to operational amplifiers. Depending on the gain associated with the particular amplifier input, the amplifier input will have an impedance of 1 MΩ, 0.2 MΩ, or 0.1 MΩ. This loads the pots and introduces inaccuracies unless the follow-up pot is similarly loaded. Thus the load resistor must be added. All potentiometer wipers on one servomultiplier must go to the same impedance input.

The symbol in Figure 19 is common for the servomultiplier.

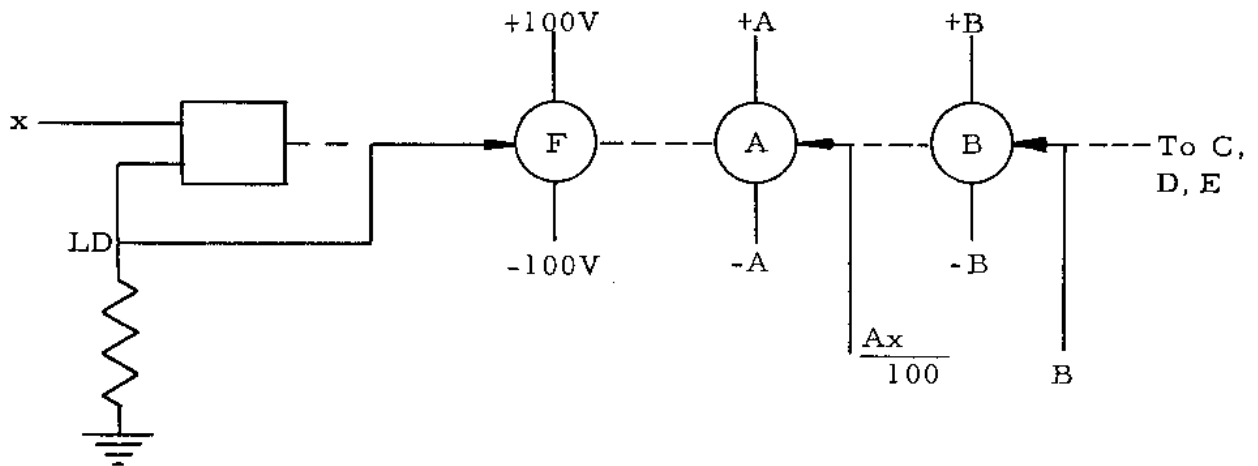
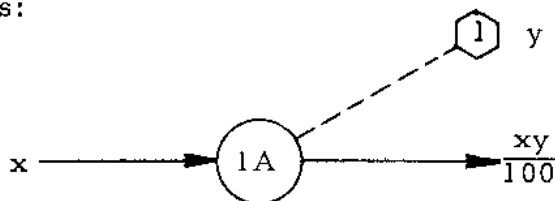


Figure 19

Servomultiplier symbol

When considering the multiplication of two voltages (which are functions of time), the slower varying voltage must be used to drive the servo, for obvious reasons. If both voltages are rapidly varying, the servomultiplier is ruled out and other types used. There are the quarter-square multiplier, which use the relation  $(x+y)^2 - (x-y)^2 = 4xy$ , and the electronic multipliers, which produce square waves with base proportional to  $x$  and height proportional to  $y$ . In the electronic multiplier, the area under the square wave is proportional to  $xy$  and the output regulated so that for  $x$  and  $y$  as inputs,  $-xy/100$  is the output.

A common space-saving symbol for servomultiplication in large flow diagrams is:



where  $\textcircled{1}$  represents the input to the multiplier and the number of the multiplier, and  $\textcircled{1A}$  represents the multiplying potentiometer of the multiplier with its appropriate connections. The arrows represent the inputs and output of the multiplying potentiometer.

Before an electronic multiplier can be used, it must be balanced. This procedure is discussed in Scott.<sup>(3)</sup>

### 5. Servodivider

Division can be accomplished by means of a servomultiplier and one additional amplifier, as shown in Figure 20, or by means of an electronic divider.

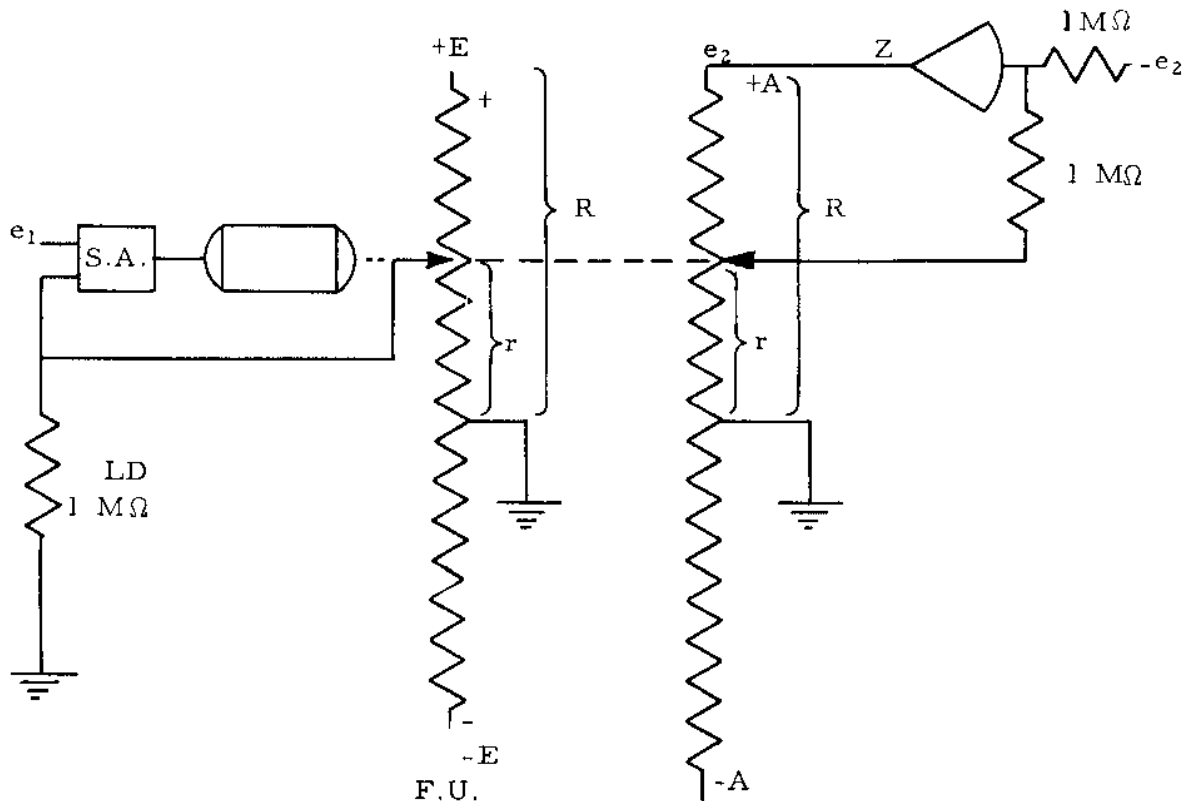


Figure 20

Servodivider circuit

In Figure 20, the resistor LD is included to indicate the loading of the F.U. potentiometer to equal the loading of cup A.

It is noted that the circuit in Figure 20 will not suffice if  $e_1$  changes sign. (negative is  $< 0$ ).

The following equations apply to Figure 20:

$$r/R = e_1/E$$

and

$$Y = (e_1/E)Z$$

The relationship between  $Z$  and  $e_2$  can be derived by considering the part of the circuit containing the high-gain amplifier (see Figure 21).

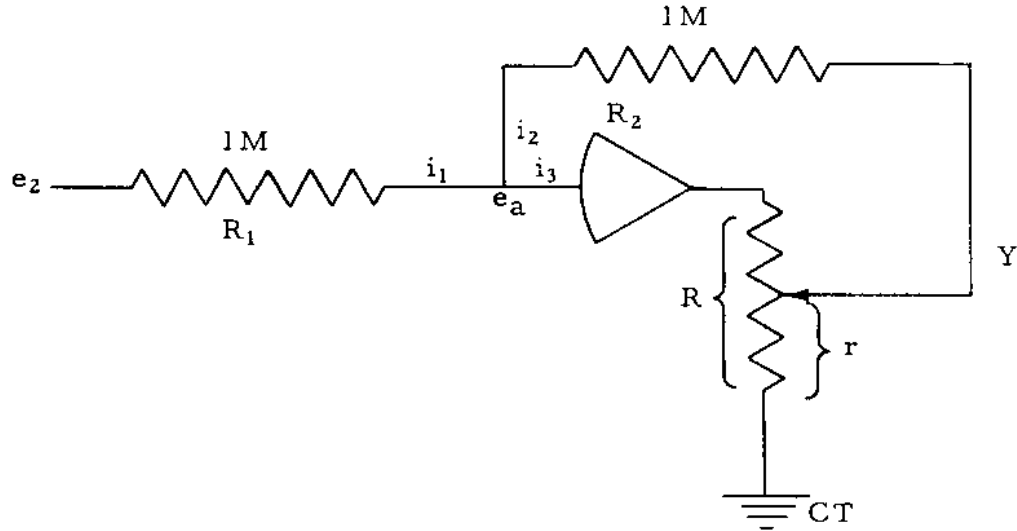


Figure 21

High-gain amplifier in servodivider circuit

$$\begin{aligned} i_1 &= i_2; & i_3 &\approx 0 \\ i_1 &= (e_2 - e_a)/R_1 \\ i_2 &= (e_a - Y)/R_2 \\ Z &= -e_a A & (A &\approx 10^8) \end{aligned}$$

$$\frac{e_2 + \frac{Z}{A}}{R_1} = \frac{-\frac{Z}{A} - Y}{R_2}$$

Since

$$R_1 = R_2$$

then

$$e_2 + \frac{Z}{A} = -\frac{Z}{A} - Y$$

But

$$Y = (e_1/E) Z;$$

then

$$e_2 + \frac{Z}{A} = -\frac{Z}{A} - \frac{e_1}{E} Z$$

so that

$$Z = \frac{e_2}{-\frac{Z}{A} - \frac{e_1}{E}} = -\frac{e_2}{e_1} E,$$

where E is usually 100 volts.

The division circuit is sometimes symbolized in analog computer diagrams in the manner indicated in Figure 22.

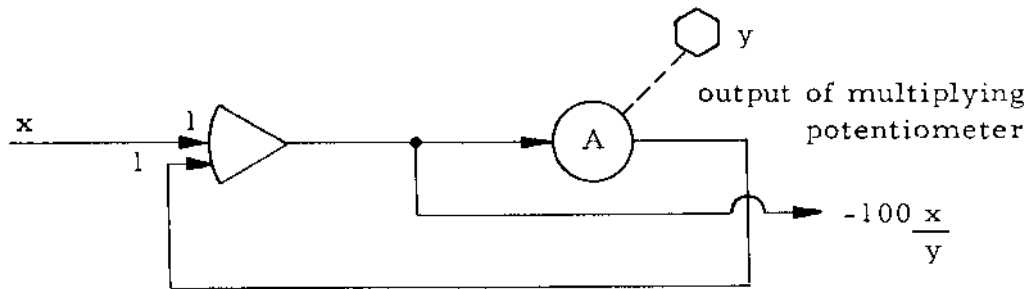




Figure 22

Divide circuit

Here  represents the servo input, and  represents the multiplying potentiometer of the servo and its appropriate connections.

This convention is adopted as a space-saving technique for use in flow diagrams where a large number of analog computer components are being used (see experiment 4).

The foregoing discussion also indicates a method for getting large gain from a summing amplifier. In Figure 23, let

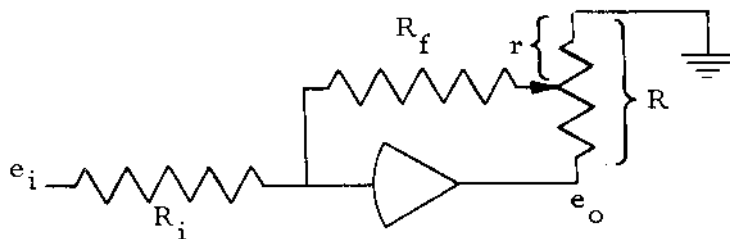


Figure 23

Large overall gain from a summing amplifier.

$$\frac{r}{R} = a < 1$$

Then

$$\frac{e_0}{e_i} = -\frac{R_f}{R_1} \frac{1}{a}$$

## 6. Function-generating Equipment\*

### a. Diode Function Generator

This unit generates  $\pm f(x)$  for an input of  $x$  by means of straight line segments. The straight line segments are produced by a network of biased diodes.

### b. Potentiometer Padder

Consider a potentiometer with many taps into which adjustable voltages are injected. This potentiometer is driven by a servo-mechanism and adjusted so that  $f(x_i)$  appears on the wiper for  $x_i$  as an input.

### c. Curve Tracker

This is used in conjunction with a plotting table. The function to be generated is drawn on 10 x 15-in. paper and a thin wire taped over the curve. A signal is fed through this wire and is detected by a pickup that is placed in the pen holder. As the arm of the plotting table is driven by  $x$ , the pickup follows the curve and the output of the table is  $f(x)$ .

## 7. High-speed Differential Relays

High-speed switching devices are needed in analog computers. The PACE computer at Argonne contains 20 such relays (10 in each console). These relays (see Figure 24) use a comparison network and a high-gain amplifier (with limiter) to effect high-speed switching.

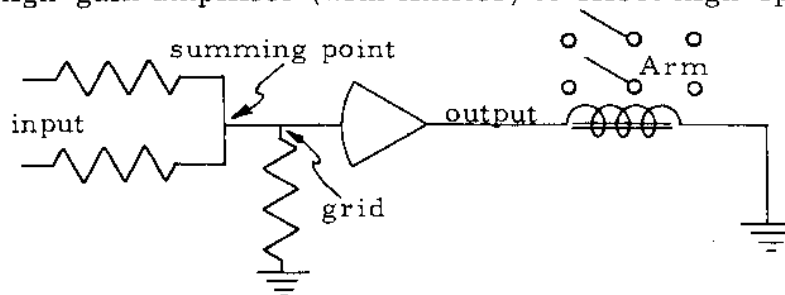


Figure 24  
Differential relay

\* A complete discussion of each of these units is contained in Scott.(3)



When the output of the high-gain amplifier is  $\left\{ \begin{array}{l} \text{negative} \\ \text{positive} \end{array} \right\}$  (and the summing point  $\left\{ \begin{array}{l} \text{positive} \\ \text{negative} \end{array} \right\}$ ), the relay arm is in the  $\left\{ \begin{array}{l} - \\ + \end{array} \right\}$  position. To use the relay, a control voltage is compared with a voltage of opposite polarity. The position of the relay is thus determined by the polarity of the larger voltage.

A common control circuit for the differential relay (see Figure 25) is an integrator and a fixed voltage. In Figure 25 the relay would trip 5 seconds after the integration starts.

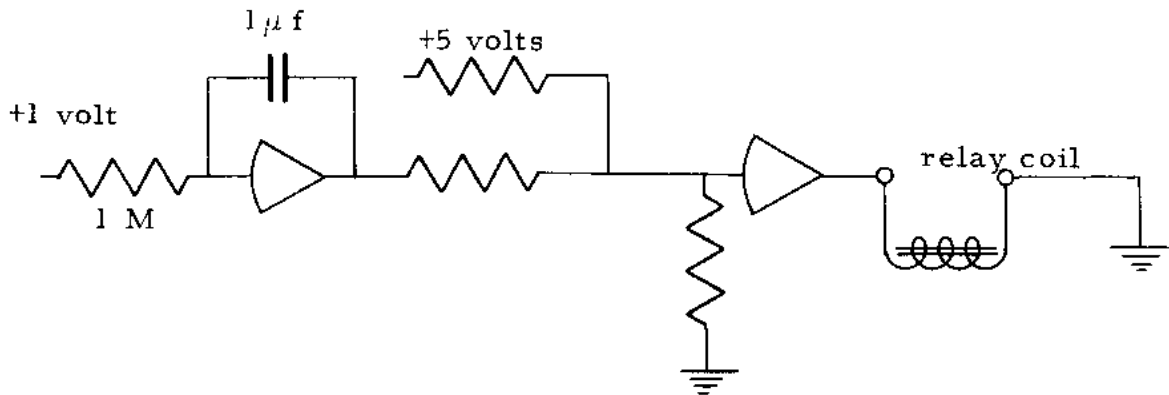


Figure 25

Typical control circuit for differential relay

## II. USE OF THE PACE COMPUTER

### A. The Patch Board

All the equipment described in Section I is located in standard relay racks. Output and input leads are wired from each component to one of 1800 gold-plated contacts of the patch bay. Patch boards (aluminum plates containing 1800 holes) are placed in the patch bay. Since a wire or lead placed into any of the holes touches one and only one contact of the patch bay, components can be interconnected to solve problems.

Figures 26 to 35 show sections of the patch panel and the connections which exist between the corresponding patch bay points and the computer components.

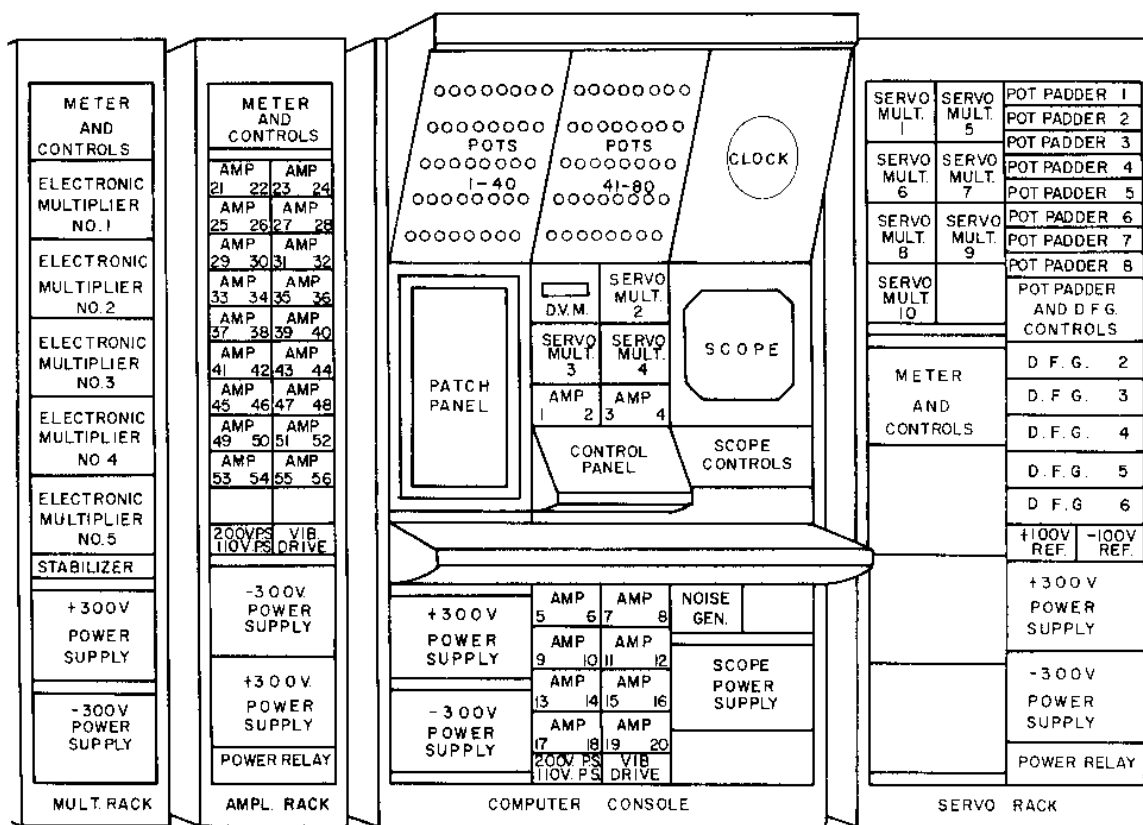


Figure 26

Component layout

Note: Page 29 (patch board graphic) can be linked to and viewed from the start page. It is too large a graphic file to include here.

The first 28 amplifiers are called combination amplifiers since they can be connected as summers or integrators. Twenty holes in the patch panel (and twenty contacts in the patch bay) are reserved for each of the first twenty-eight amplifiers.

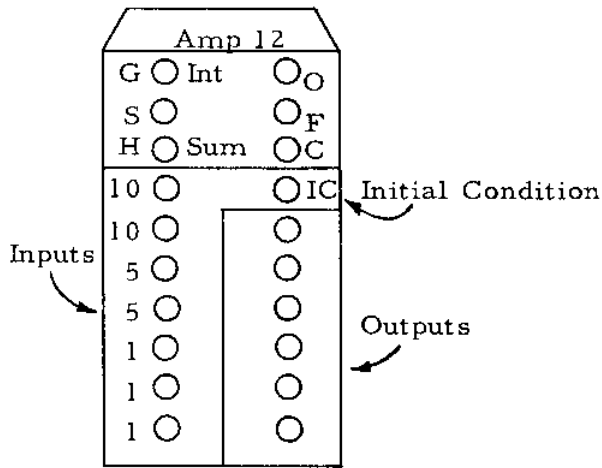


Figure 28

Patch board connections for integrating amplifier

If, in Figure 28, G is connected to S and O to F, then the amplifier is a summer. If S is connected to H and F to C, then the amplifier is an integrator. Special plugs are available for this purpose. These plugs allow INT to show when connecting as an integrator and SUM when connecting as a summer.

The connections to contacts of the patch bay are shown in Figure 29, with the contacts shown in their actual physical orientation at the computer.

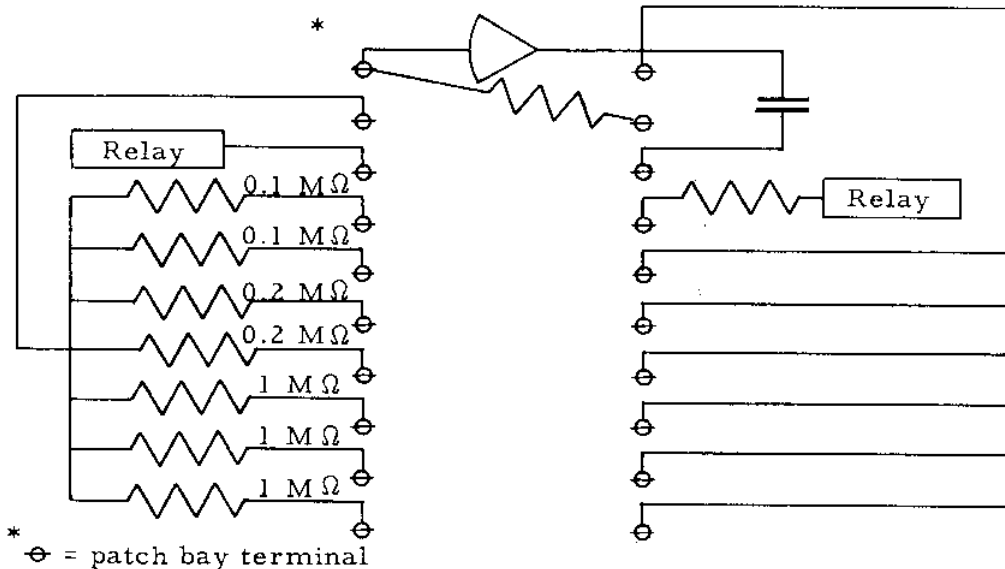


Figure 29

Patch bay connection for integrating amplifier

Figure 30 shows the same information as in Figure 29, but the representation is a more obvious electrical schematic diagram.

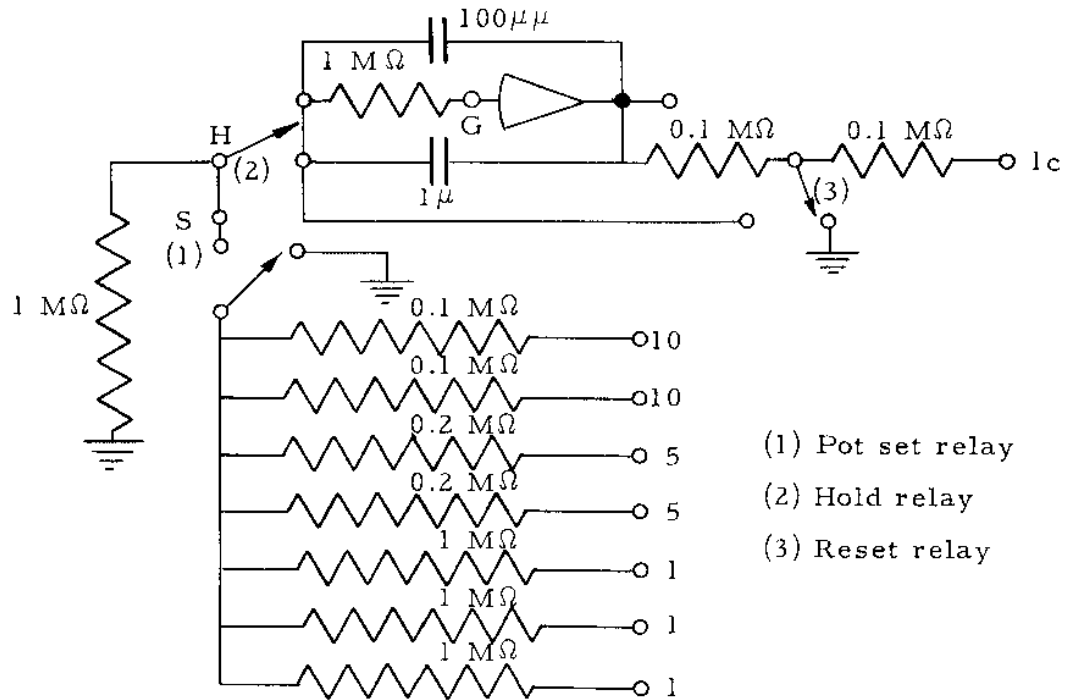


Figure 30

Integrating amplifier circuit

Amplifiers 29 to 56 are only used as high-gain amplifiers or summers. The following drawings represent the section of the patch panel and an electrical schematic of the patch bay connections.

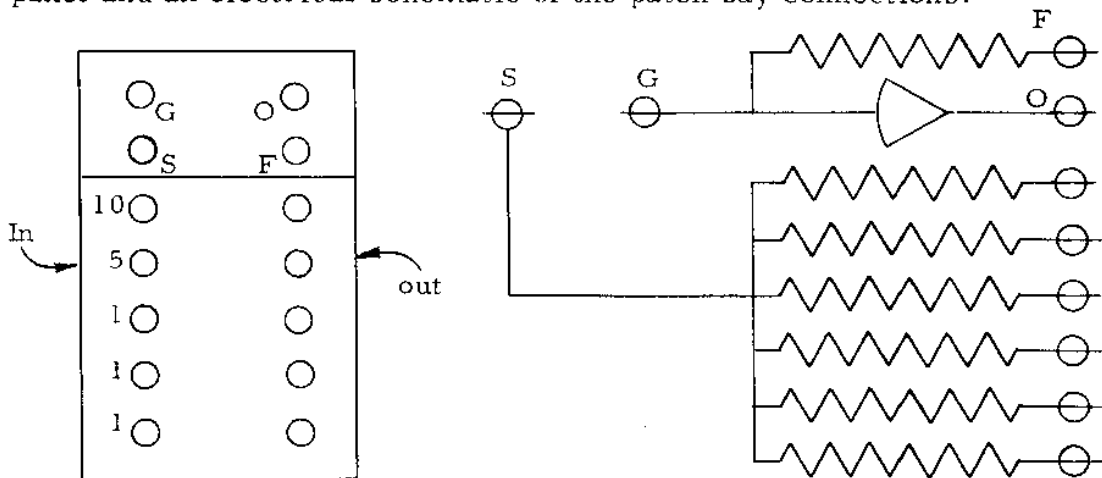
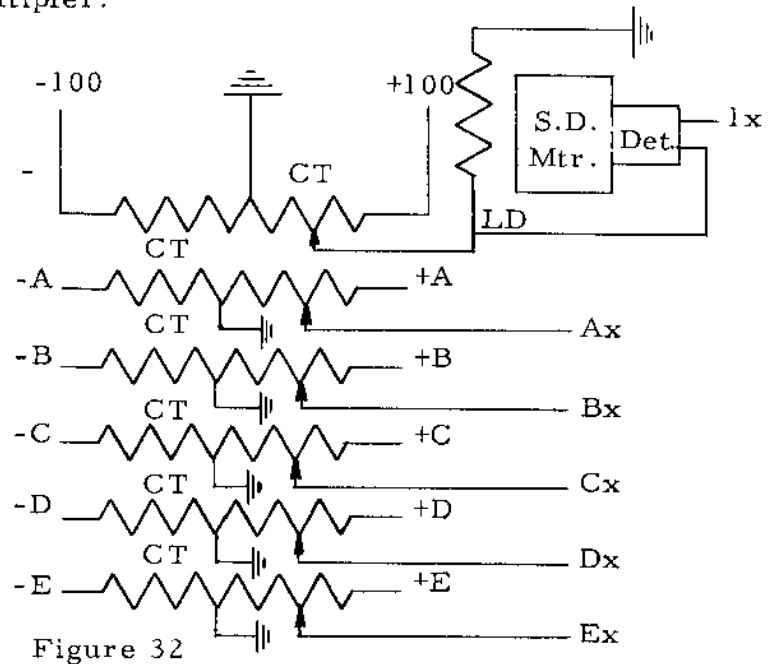
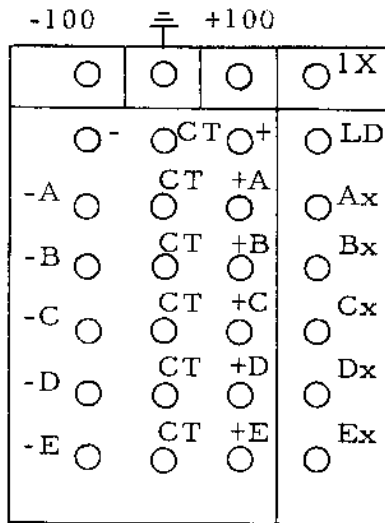


Figure 31

Summing amplifier

The section of the patch board reserved for a single servo-multiplier has 28 holes. The following drawings represent the patch board region for a single servo-multiplier and the schematic diagram of the patch bay terminals to the elements of the multiplier.



Servomultiplier connections

As indicated in the section on servo-multiplier theory, the wipers (AX, BX, CX, DX, and EX) must each connect to identical gain operational amplifier terminals. The load resistor is connected to LD on the patch board and is 1M, .2M or .1M when the gain is 1, 5 or 10 respectively.

Figure 33 shows the physical appearance of the patch board connection for a potentiometer and the electrical schematic of the patch bay connections to the potentiometer.

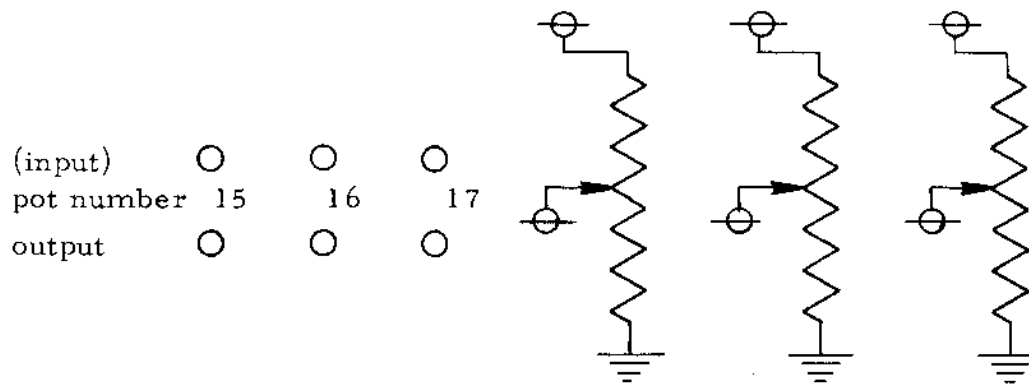


Figure 33

Potentiometer connections

The high-speed differential relays (see Figure 34) have ten connections to the patch bay.

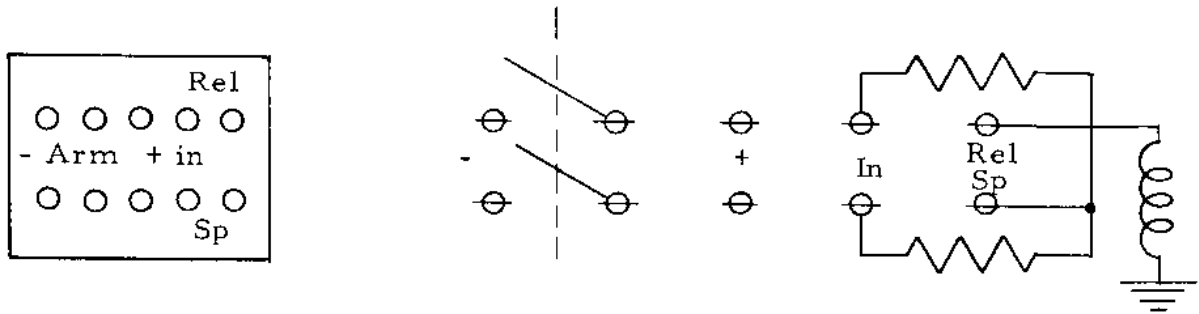


Figure 34

### Differential relay connections

The patch panel has provisions for connecting to six plotting boards (see Figure 35). The x input drives the pen and the y input drives the arm. Voltages may also be patched directly into a Sanborn six-channel recorder (the Argonne facility is equipped with a four-channel recorder).

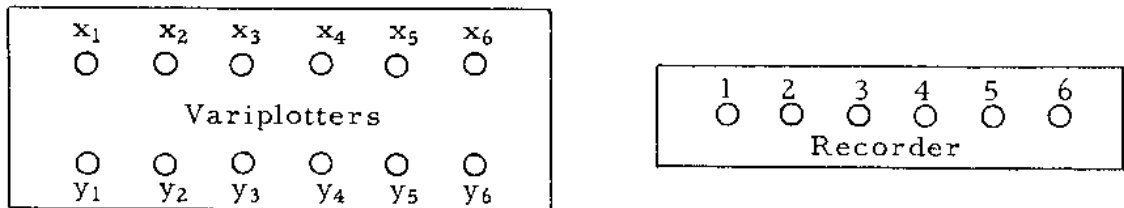


Figure 35

### Recorder connections

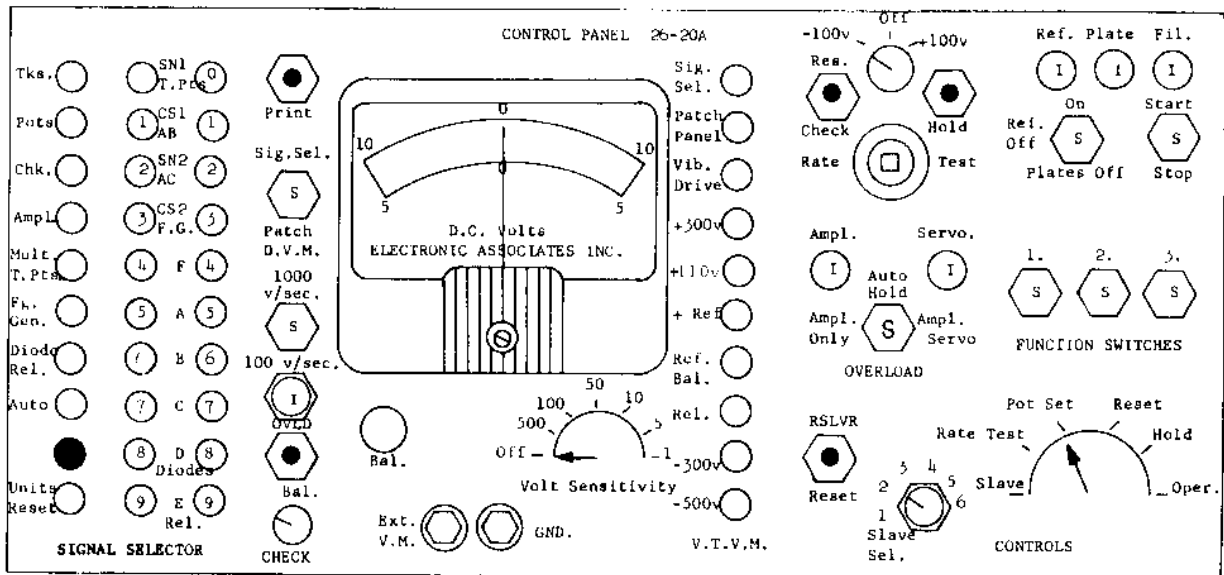
In addition to the foregoing patch bay terminals, the following components are connected to terminals in the patch bay:

- |                               |                        |
|-------------------------------|------------------------|
| diode function generators     | VTVM                   |
| diodes                        | hold relay control     |
| +100 volts                    | selector switch        |
| -100 volts                    | electronic multipliers |
| high-quality ground           | DPDT switches (called  |
| boost resistors (plugged into | function switches)     |
| outputs of amplifiers that    |                        |
| are driving large loads)      |                        |
| digital voltmeter             |                        |

## B. The Selector Switch

The selector switch on the computer control console (see Figure 36) is used to select voltages in the machine for measurement on a voltmeter (digital or VTVM). The selector switch and digital voltmeter are also used for setting potentiometers.

When a wired patch board is set into the patch bay, the potentiometers used in the circuit must be set. Some potentiometers are used to inject constant voltages (initial conditions, forcing functions or control voltages) into the system, while others are used to provide coefficients for problem variables. Twenty potentiometers are provided for the first use and sixty are provided for the latter.



- I - Indicator Bulb
- S - Switch

Figure 36

Control panel diagram

The voltage  $e_0$  on the wiper arm of a potentiometer is of interest either for itself or because for known  $e_i$  the relation  $e_0 = (r/R)e_i$  can be used to determine the ratio  $r/R$ . The voltage  $e_0$  may be read on the digital voltmeter, but a different procedure is required for setting coefficient and initial condition potentiometers.

Beneath each initial condition potentiometer (1-20) is a 3-position switch (see Figure 37). This switch can be used to connect the input of the pot to -100 v, patch board (PB), or +100 v.

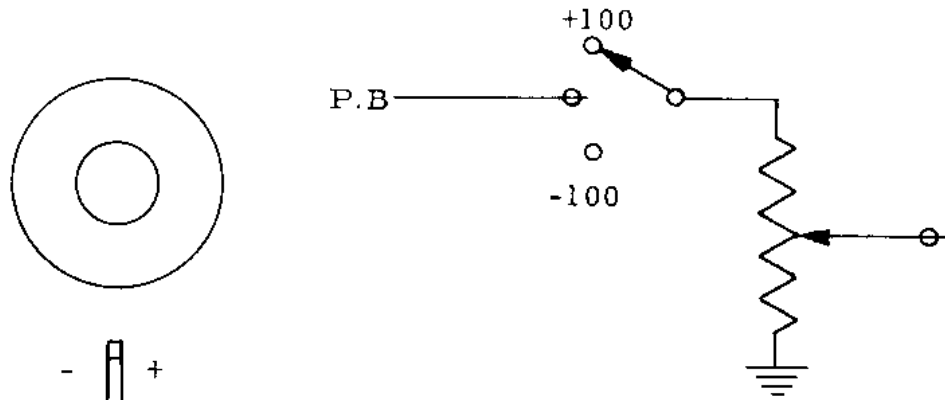


Figure 37

### Initial condition potentiometer

To read a voltage or ratio for potentiometers 1-20, the following switch setting must be used: Push-button signal selector switches

First row - pot

Second row - first digit

Third row - second digit (pot 20 = 00)

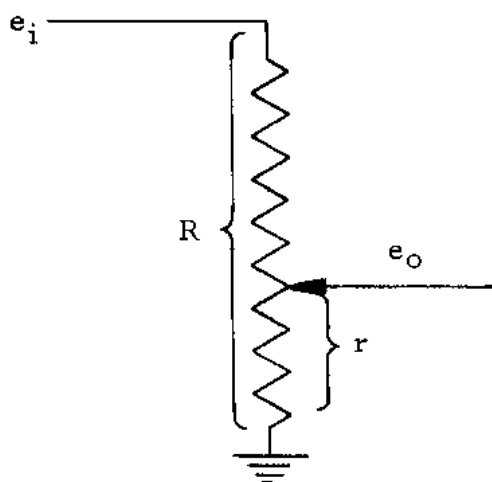
Digital Voltmeter Switch DVM - Signal selector position

Vacuum Tube voltmeter (VTVM) - Patch panel position

Example (see Figure 38):

Suppose pot 5 is to supply an initial condition of 5 volts to some integrator. We select pot 05 and put the switch under pot 5 into the + position. Remember that this puts +100 volts on the input of pot 5.

Adjust the pot until the digital voltmeter reading is 0500. This means the arm is set so that  $e_o = 5$  volts. From



$$e_o = (r/R) e_i.$$

If  $e_i = 100$  volts, then

$$r/R = 0.05.$$

Figure 38

Example of potentiometer setting



Coefficient potentiometers 21 to 80 also have a 3-position switch located directly beneath them, (see Figure 39). The switch is spring loaded to be in a central position. In the coefficient position, +100 v is placed on the input to the pot and the DVM is connected to the arm. NOTE: This overrides whatever was selected on the Signal Selector Switch.

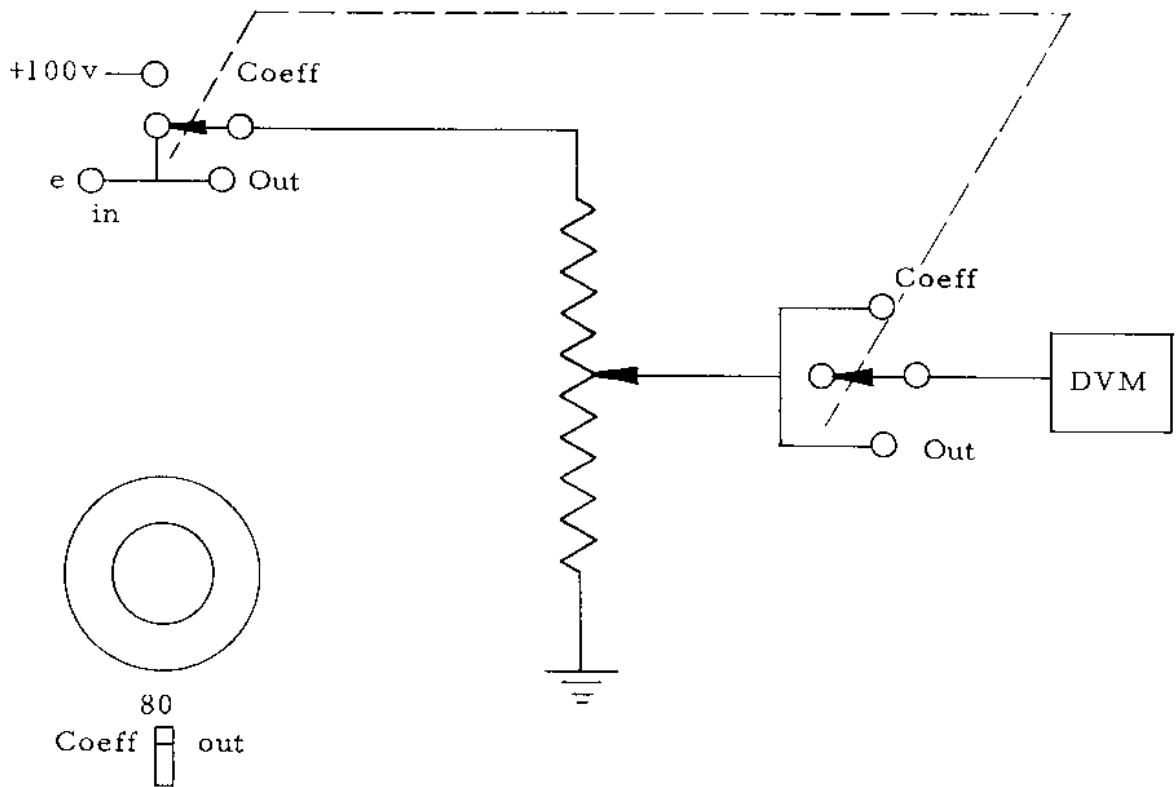


Figure 39

Coefficient potentiometer

If the output of the DVM is read as a four-place decimal,

$$e_0 = ae_j,$$

where a is the reading of DVM.

In the output position, the actual output of the potentiometer is read with an input produced by the patched problem. In this case the DVM is read with the decimal between the second and third figures.

The output of any amplifier may be monitored by depressing buttons amp, first digit, and second digit in the three rows of push buttons on the selector switch.

To read the output of any servo-multiplier the push buttons are: Multiplier, Number of Multiplier, Letter of cup.

Diode Function generators may be monitored by selecting Function Generator, Number of Unit, FG.

In the bottom row of the patch board, there is a hole marked SEL. Whatever voltage is selected on the selector switch appears here. Since the solution for a problem generally involves graphs of many voltages, this feature can be used to simplify the plotting of results (see Figure 40).

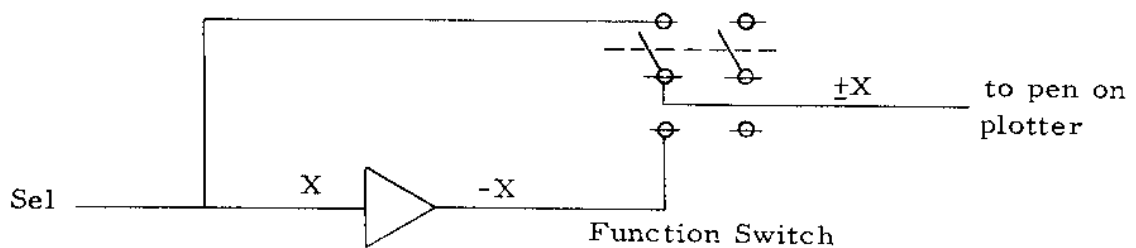


Figure 40

Use of selector switch in plotting

### C. The Operational Controls

Since time is the independent variable in an analog computer, there must be a method for starting a problem at  $t = 0$  and stopping it at  $t = t'$ . An operational selector switch makes this possible, along with other modes of operation. This switch controls relays in the machine and has 6 positions:

- 1) Slave - used when the computer is being controlled by another. Up to six computers may be slaved together.
- 2) Rate test - used to check the feedback capacitors for the integrators.
- 3) Pot set - In this position, the summing junction S is grounded and disconnected from the grid G. Thus amplifiers cannot be overloaded while pots are being set, and pots are set while connected to their operative load.

- 4) Reset - Two cases must be considered here, amplifiers and integrators.
  - a. Integrators - This position disconnects H and C and applies the initial conditions across the feedback capacitor. Thus time is held at zero and the output of the integrator is equal to the initial condition.
  - b. Summing amplifiers have G and S connected, F and O connected.
- 5) Hold - By placing the control in the hold position a computation is interrupted and time stands still. The output voltages of all amplifiers remain fixed at the values they had when the computation was interrupted. H is disconnected from C and initial conditions are not applied on integrators. Summing amplifiers are connected as in reset.
- 6) Operate - In the operate position all computer elements are connected and active in the solution of the problem. For integrators H is connected to C, initial conditions are removed, and the feedback capacitor is connected across the high-gain amplifier. Summing amplifier connections are unchanged from the reset position.

#### D. Special Circuits

During the course of these experiments the need will arise for the following circuits:

- 1) Relay circuit controlled by time (see Figure 41). The relay would flip "a" seconds after the integrator started.

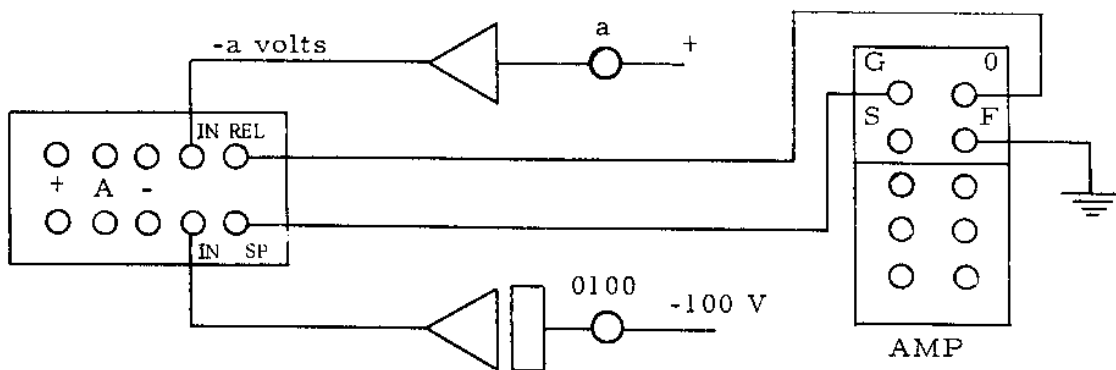


Figure 41

Time controlled relay

- 2) Relay controlled by a problem voltage, as in Figure 42.

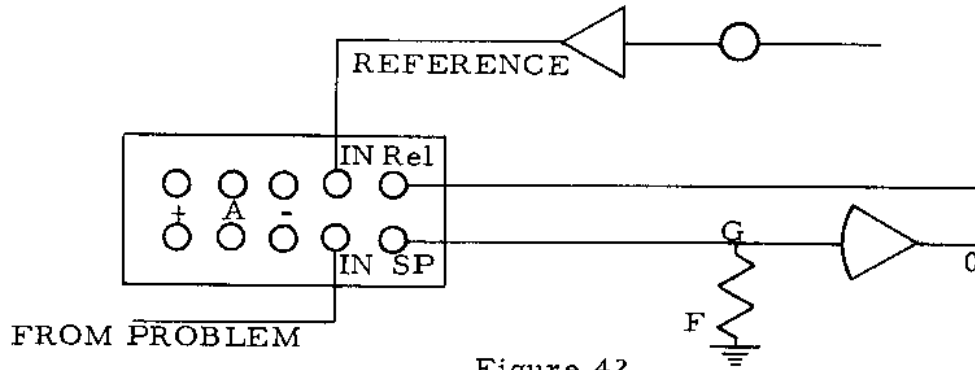


Figure 42

Relay controlled by machine variable

- 3) Square Wave generator controlled by a sine-wave generator, as in Figure 43.

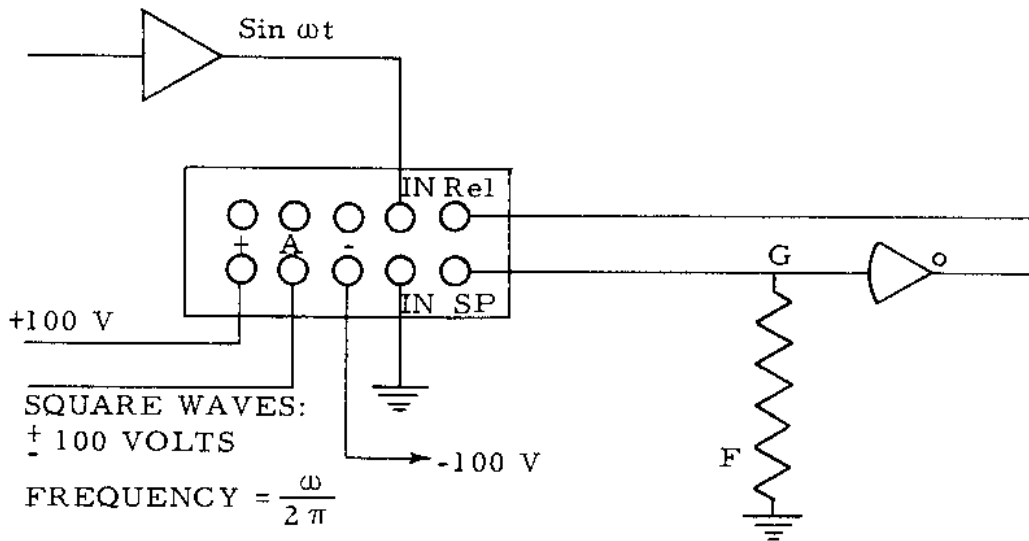


Figure 43

Square-wave generator

- 4) Square-wave generator (symmetrical or asymmetrical) as in Figure 44.

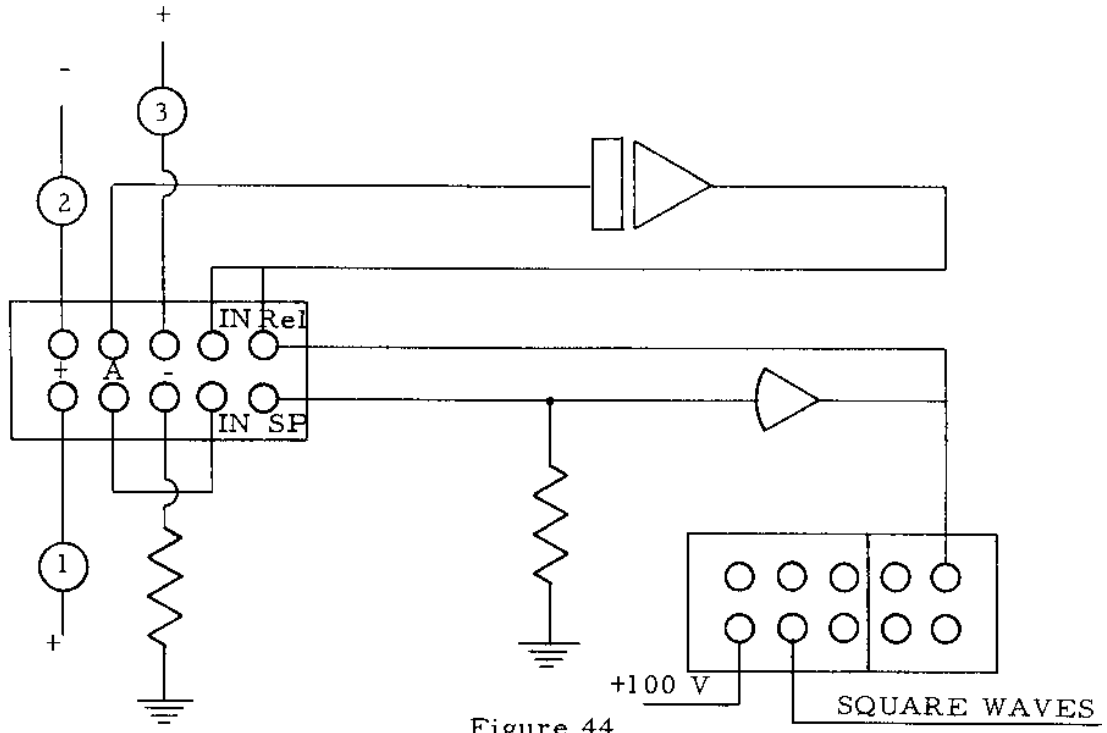


Figure 44

Square-wave generator

5) Amplifier with constant gain  $> 1$  (see Figure 45).

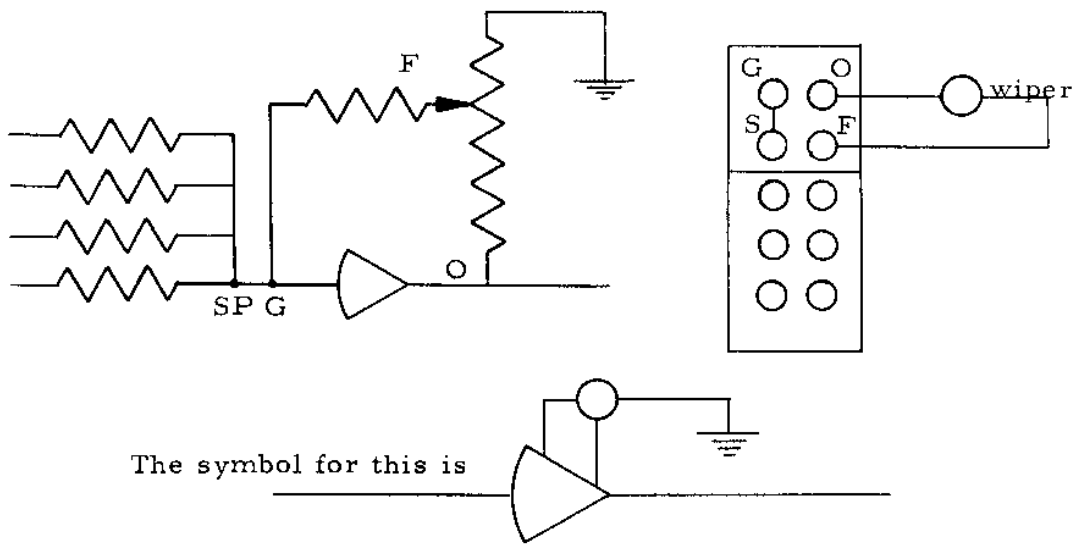


Figure 45

Gain multiplier for summing amplifier

6) Use of two Input networks with one amplifier (see Figure 46).

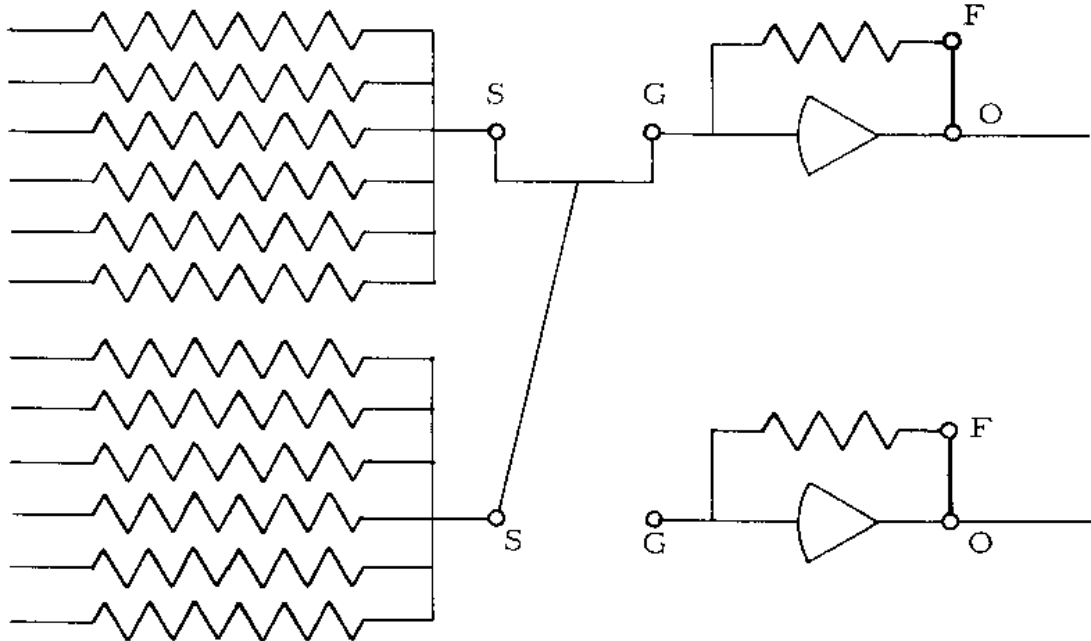


Figure 46

Use of two input networks with one amplifier

(NOTE: All outputs must be taken from the top amplifier.)

7) Amplifier gain reduction, as in Figure 47.

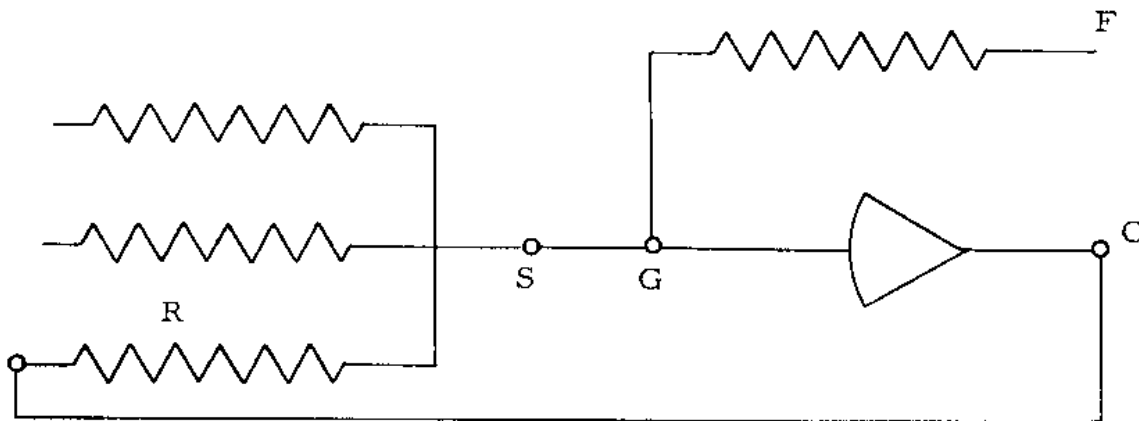


Figure 47

Gain reducer for summing amplifier

(If  $R = 0.1 \text{ M}\Omega$ , the gain is  $1/10$  and if  $R = 0.2 \text{ M}\Omega$  the gain is  $1/5$ .)

8) Automatic Hold circuit.

In the bottom row of the patch board are holes marked 60 v and hold. If these are patched together the hold relays are energized. The circuit in Figure 48 allows the problem to be put into hold when the control conditions are satisfied.  $R_1$  has to be patched in because this circuit bypasses a  $500 - \Omega$  dropping resistor in the hold relay circuit.

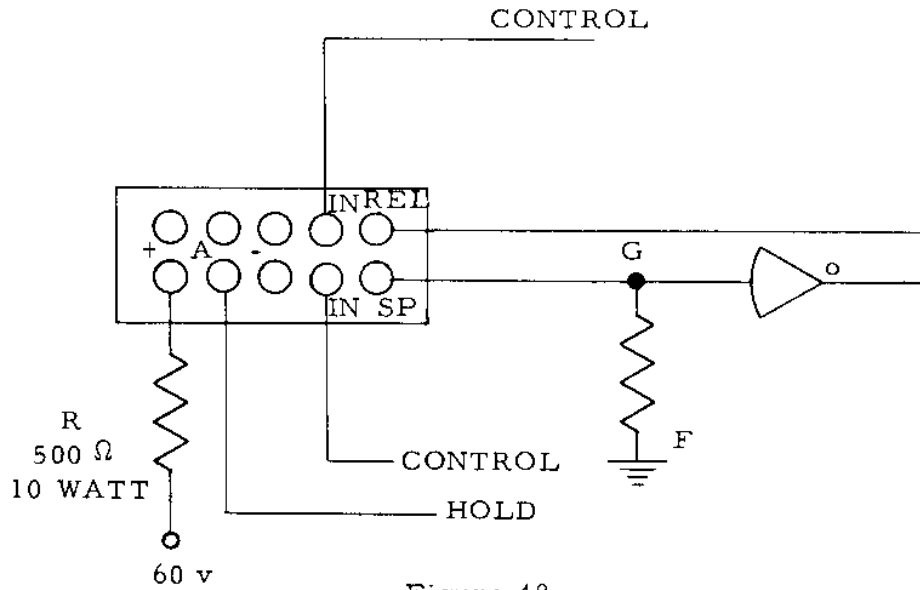


Figure 48

Automatic hold circuit

### III. PROGRAMMING FOR AN ELECTRONIC ANALOG COMPUTER

#### A. Definition and Illustration of Programming

Programming may be defined as the sum of the operations necessary to prepare a problem for the analog computer. It includes the following operations:

1. Statement of the problem and any modifications that will aid in securing the best results from the computer.
2. Assignment of scale factors to each problem variable. This is done to keep voltages (the analogs of the variables) between + and - 100 volts, to stay within the limits of the computer, and as high as possible for greatest accuracy. If X is a problem variable, the relation  $X' = aX$  defines a machine variable X' with scale factor a.
3. Preparation of scaled equations by substituting the new variables into the original equations.
4. Preparation of machine equations by substituting values of constants into scaled equations.
5. Preparing a circuit diagram and a sheet showing potentiometer settings.
6. Connecting the components together on the patch board.
7. Setting the patch panel into the patch bay.
8. Setting pots, static checking, and debugging.
9. Getting results and checking solutions.

It is best to take a simple problem with a known solution for an example. Thus assume the equation to be "solved" is

$$\frac{d^2y}{dt^2} = -\omega^2 y \quad .$$

The solution of this differential equation is  $y = \sin \omega t$ .

In order to make programming clear it is best to restate the problem. Thus it is required to generate  $y = \sin \omega t$  rather than to solve  $d^2y/dt^2 = -\omega^2 y$ . The differential equation will be worked with later.



The function  $y = \sin\omega t$  has an amplitude of unity and a frequency of  $\omega/2\pi$ . Since it is best to work with large voltages, a scale factor  $b$  will be assigned to  $y$ . In this manner define the machine variable  $y' = by$ . If  $b = 10$ , the sine waves generated will have an amplitude of 10 volts.

There are times when it is desirable to slow down or speed up time. This is done by defining  $t' = at$ . If  $a > 1$ , one second of actual time will be represented by  $a$  seconds in the computer. If  $a < 1$ , then one second on the computer represents more than one second in actual time.

Starting with  $y = \sin\omega t$  and substituting

$$y' = by$$

$$t' = at$$

the scaled equation

$$y' = b \sin \frac{\omega t'}{a} \text{ is derived.}$$

If  $y'$  is differentiated twice, there are obtained the equations

$$\frac{dy'}{dt'} = \frac{\omega b}{a} \cos \frac{\omega t'}{a}$$

and

$$\frac{d^2 y'}{dt'^2} = \frac{-\omega^2 b}{a^2} \sin \frac{\omega t'}{a}$$

or

$$\frac{d^2 y'}{dt'^2} = \frac{-\omega^2 y'}{a^2} .$$

The initial conditions are as follows:

$$\left. \frac{dy'}{dt'} \right]_{t=0} = \frac{b\omega}{a}$$

$$\left. \frac{d^2 y'}{dt'^2} \right]_{t=0} = 0 .$$

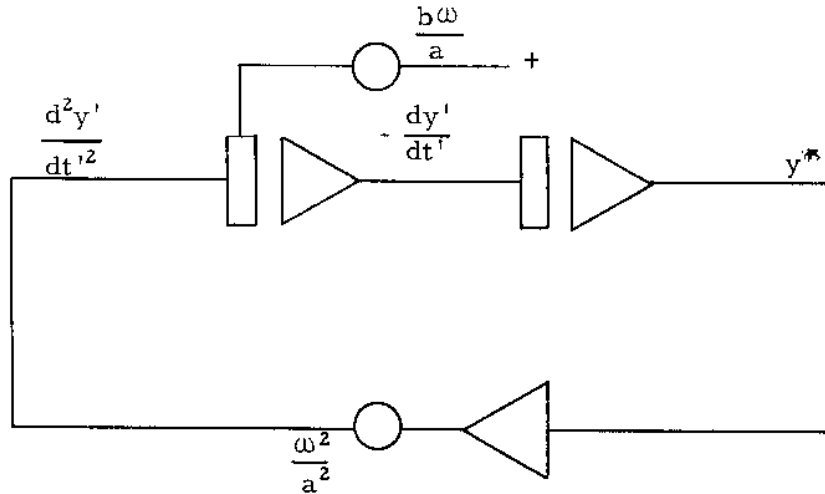


Figure 49

Sine-wave generator

In the circuit of Figure 49, the initial condition changes with frequency. In Figure 50 is a circuit in which the initial condition is independent of frequency. Instead of using the equation

$$\frac{d^2y'}{dt'^2} = \frac{-\omega^2y'}{a^2} \quad ,$$

we use the form

$$\frac{a}{\omega} \frac{d^2y'}{dt'^2} = \frac{-\omega y'}{a}$$

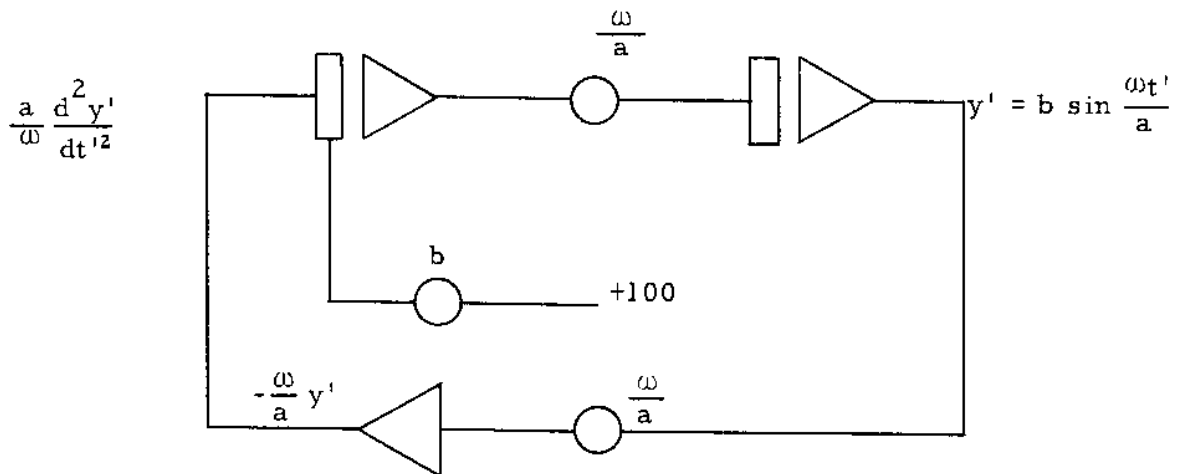


Figure 50

Sine-wave generator

In order to determine if a system has been wired on the patch panel correctly, the analog programmer sets up what is commonly referred to as a "Static Check."

After a problem is placed on the machine, and all coefficient potentiometers set, a known voltage is impressed on the Initial Condition of the integrating amplifiers. This voltage (inverted in sign) will appear at the output of the integrator.

By tracing through the system we can calculate what the output voltage of each component should be, with the master control switch in the reset position. If it is determined that all outputs are of correct magnitude and sign, it can be safely assumed that the problem is wired correctly.

Since the inputs to the integrating amplifiers are not connected to the amplifier grid in the reset position of the master control switch, the input voltages to the integrators will not appear at the amplifier output.

It is possible that a system will static check properly and the dynamic solution be incorrect. Rectification of such a problem will be discussed in experiment number 4.

#### B. Aids to Scaling and the Formulation of Machine Equations

In the previous section, scale factors were applied to the solution and scaled equations were developed. Of course, this is not generally the case. For if the solution is known, what is the problem?

It is more usual to start with the equations to be solved and make the following substitutions:

$$y' = by$$

$$t' = at$$

$$dy' = bdy$$

$$d^2y' = bd^2y$$

$$dt' = adt$$

$$dt'^2 = a^2dt^2$$

If these are substituted into

$$\frac{d^2y}{dt^2} = -\omega^2y \quad ,$$

the scaled equations become

$$\frac{\frac{d^2 y'}{dt'^2}}{a^2} - \frac{\omega^2 y'}{b}$$

or

$$\frac{d^2 y'}{dt'^2} - \frac{\omega^2 y'}{a^2} .$$

These are identical with the scaled equation in the previous section.

---

## IV. EXPERIMENTS

### A. Sine-wave Generator

1. Equation:

$$\frac{1}{\omega} \frac{d^2 y}{dt^2} = -\omega y$$

2. Scale Factors and Constants:

$$y' = by \quad 0 < b \leq 100$$

$$t' = at \quad a > 0$$

$$\omega = 2\pi f$$

3. Scaled Equations and Initial Conditions:

$$\frac{a}{\omega} \frac{d^2 y'}{dt'^2} = -\frac{\omega y'}{a}$$

$$t = 0 \quad ; \quad \frac{a}{\omega} \frac{dy'}{dt'} = b \quad ; \quad \frac{d^2 y'}{dt'^2} = 0 \quad .$$

4. Circuit Diagram (given in Figure 51)

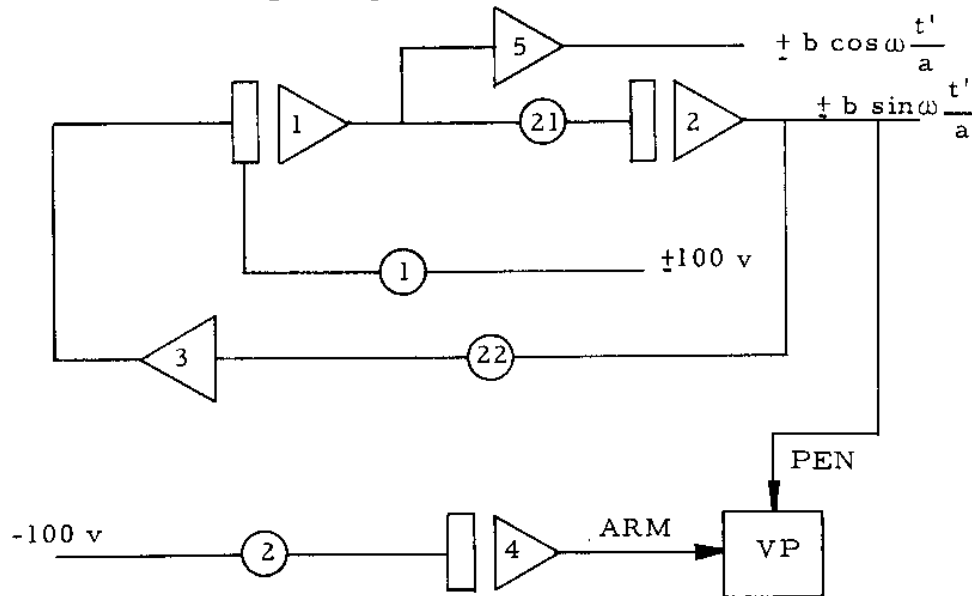


Figure 51

Complete analog diagram for solution of  $\frac{d^2 y}{dt^2} = -\omega^2 y$

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**5. POTENTIOMETER SETTINGS**

PROBLEM NO. \_\_\_\_\_  
 DRAWING NO. \_\_\_\_\_  
 DATE \_\_\_\_\_

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORRECTION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
1		+ b volts	+10		1000		If b = 10 $\omega = 6.283$ a = 1 A sine wave of 1 cps is generated
2		- 1 volt	-1		0100		
21		$\omega/a$	6.283		6283(10)		
22		$\omega/a$	6.283		6283(10)		

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**6. STATIC CHECK**

PROBLEM NO. \_\_\_\_\_  
 DRAWING NO. \_\_\_\_\_  
 DATE \_\_\_\_\_

UNIT	UNIT NUMBER		OUTPUT (VOLTS)	REMARKS	INTEGRATOR	INITIAL CONDITION	SET	PARAMETERS
	DRAWING	MACHINE						
Amp	1		-10		1	-10.00		
	2		0					
	3		0					
	4		0					
	5		+10					
Pot	1		+10					
	21		- 6.28					
	22		0					
	2		- 1.0					

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Note: It was decided to end the conversion of the manual to pdf format at this point. The original manual has thirty more pages of examples relating to nuclear engineering, and there is a companion manual of engineering applications.