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AN ANALOG COMPUTER MODEL OF A MULTIPLE REGION REACTOR

by

L. C. Just, C. N. Kelber,
and
N. F. Morehouse, Jr.

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AN ANALOG COMPUTER MODEL OF A MULTIPLE-REGION REACTOR

by

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AN ANALOG COMPUTER MODEL OF A MULTIPLE-REGION REACTOR

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ABSTRACT

This paper presents a technique for solving the time-space-dependent diffusion equation for a multiple-region reactor on an analog computer. The technique was applied to a reactor problem involving fuel burnup and poison buildup, while constant power was maintained by the introduction of control poison.

Diffusion across boundaries is computed by putting the interregion neutron diffusion into the form of a surface integral. This method allows a good flux plot to be made by means of only six spatial points.

A sample problem was worked by this technique and compared to a digital solution. All computations, circuits, and operating information necessary to duplicate the experiment are given. Also, results are included and compared with a digital computation.

Table 1

SYMBOLS

B_z^2	axial buckling
D	diffusion coefficient
D_f	diffusion coefficient for fast group
D_s	diffusion coefficient for slow group
D_{fi}	diffusion coefficient for fast group in the i^{th} region
D_{si}	diffusion coefficient for slow group in the i^{th} region
D'_{fi}	$D_{fi}/r_{i+1} - r_i$

Table 1 (Cont'd.)

D'_{si}	$D_{si}/(r_{i+1} - r_i)$	
$\mathcal{D}_{fi}$	$2\ D'_{fi}\ D'_{fi+1}/(D'_{fi} + D'_{fi+1})$	
$\mathcal{D}_{si}$	$2\ D'_{si}\ D'_{si+1}/(D'_{si} + D'_{si+1})$	
f	subscript for fast group	
FP	fission products	
i	subscript for i^{th} region	
J	leakage coefficient (Eq. 17)	
n	neutron density	
P	power at time t	
P_0	desired power	
r	radial distance	
s	subscript for slow group	
S	neutron sources	
S_i	surface area between regions i-1 and i	
v	neutron velocity	
V_i	volume of i^{th} region	
γ	fission yield	
λ	decay constant	
λ_t	transport mean free path	
ν	fission neutrons per fission (average)	
$\Sigma_{a\,c+m}$	macroscopic absorption cross section for cladding and moderator	
Σ_{ac}	macroscopic absorption cross section for control poison	
$\overline{\Sigma}_a$	total macroscopic absorption cross section for the core	
σ_a	microscopic absorption cross section	these are further subscripted in the text to relate them to a particular element
σ_f	microscopic fission cross section	
τ	Fermi age, cm^2	
ϕ	neutron flux, n/cm^2-sec	
∇	gradient	
∇^2	Laplacian operator	

I. INTRODUCTION

The neutron flux in a reactor may frequently be represented by the time-dependent solution of the diffusion equation. When it is possible to characterize the neutron reaction rates in the reactor by their values at only a few spatial points, an analog computer of reasonable size can be used to give a detailed history of the neutron flux and reaction rates.

In reactors composed of many regions of differing composition, a common practice is to compute the flux at many spatial points and for comparatively few temporal points. Such programs of computation employ a large digital computer (e.g., the IBM-704) and would require an extremely large analog machine for their execution.

In the program presented here, spatial resolution is given up in favor of a detailed flux history. The extreme example of such computations is the one-point technique. In the one-point technique the spatial dependence of the flux is completely suppressed, and the average flux and compositions are used instead.

One-point solutions abound in the literature, but when the reactor has regions of sharply varying composition and fluxes the one-point solution is clearly inapplicable. The program presented here applies the one-point approximation in each region; appropriate boundary conditions are used to estimate neutron diffusion effects, and neutrons are conserved. Thus the extremely simple one-point method is extended and becomes applicable to a much wider range of problems. This extension is brought about with a relatively small increase in the amount of analog computer equipment used.

The equations solved are those for fuel depletion, saturating fission product buildup, poison burnup, xenon and iodine buildup and reactor control poison. The method here described has been applied to the study of a three-region reactor. Six space points were used. Figure 5 is a flow chart for this problem.

At each space point two fluxes are computed - the fast (ϕ_f) and the slow flux (ϕ_s). The method of calculating the diffusion between regions is described in Section II. In Section III, the method of computing reaction rates, conserving neutrons and calculating burnup is given.

Section IV is concerned with details peculiar to the analog computer. Section V is a presentation of the results of a specific problem, while in Section VI these results are compared with those of a comparable digital program.

II. THE TIME-DEPENDENT DIFFUSION EQUATION

Any studies of the time-dependent diffusion equation on an analog computer of reasonable size will have the following points in common:

(1) The reactor will be divided into a number of regions with provision for a solution in each region;

(2) The energy-dependence will be taken into account by dividing the energy spectrum into n ranges and solving an equation for each energy group in each region;

(3) $\nabla D \nabla \phi$ must be approximated as a function of one space variable only.

For a unit volume in a homogeneous region

$$\frac{\partial n}{\partial t} = \nabla D \nabla \phi - \Sigma_a \phi + S \quad , \qquad (1)$$

for one energy group. Since $\phi = nv$ and D is constant in each region,

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\nabla^2 \phi - \Sigma_a \phi + S \quad . \qquad (2)$$

For an arbitrary volume V,

$$\iiint \frac{1}{v}\frac{\partial \phi}{\partial t} dV = \iiint D\nabla^2 \phi \, dV + \iiint (S - \Sigma_a \phi) \, dV \quad . \qquad (3)$$

But

$$\iiint \nabla^2 \phi \, dV = \iint D \nabla_n \phi \, dS \quad , \qquad (4)$$

where $\nabla_n \phi$ is the component of $\nabla \phi$ directed toward the outward normal to the surface S.

Since the volume integral of the divergence of the neutron current can be replaced by the surface integral of the current (i.e. $\nabla_n \phi$), difficulties inherent in approximating second derivatives are avoided. This manipulation will also reduce the amount of analog equipment needed.

The solution of Eq. (3) involves an assumption as to the spatial variation of the flux within each region. The assumption used here is that there is a linear variation in the flux between the mid-point of a region and the boundary of that region.

Equation (3) is still a three-dimensional problem, but it may be reduced to one of one dimension. The reactor under study is cylindrical with three annular regions (Fig. 1); this means that the flux distribution will be a function of the radius and the height. If a cosine variation is assumed in the axial direction, a correction for axial leakage can be added to the absorption loss in each region. Therefore, the calculation of $D\nabla_n \phi$ can be done with respect to one space variable.

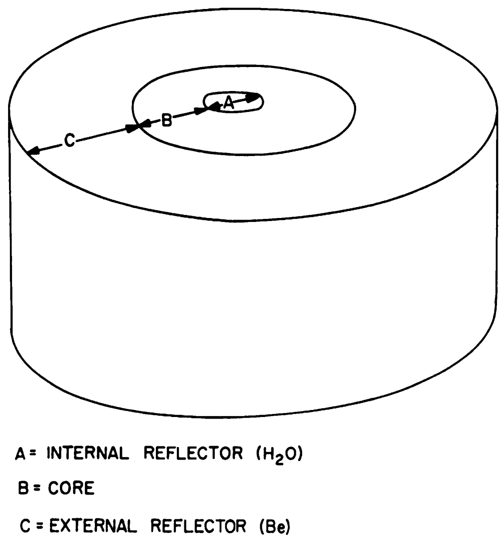


Fig. 1. Three-region Reactor

It is now possible to determine the function that will describe the diffusion between two regions. Two assumptions are made:

(1) Within a region, a linear variation in flux exists from the center to the boundary;

(2) At an interface,

(a) $\lim_{\epsilon \to 0} \phi(r+\epsilon) = \lim_{\epsilon \to 0} \phi(r-\epsilon)$

(b) $\lim_{\epsilon \to 0} D_1 \nabla \phi(r+\epsilon) = \lim_{\epsilon \to 0} D_2 \nabla \phi(r-\epsilon)$.

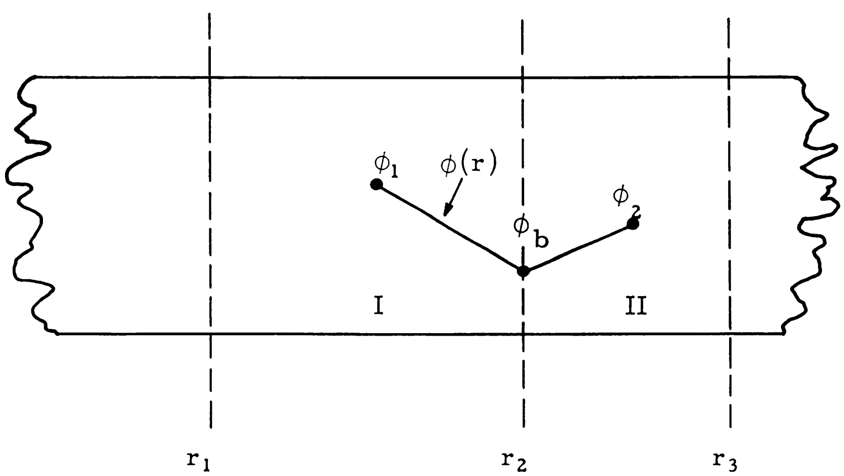


Fig. 2. Regional Flux Variation

In the right semi-region of I,

$$\nabla\phi(r) = \frac{\phi_b - \phi_1}{\frac{r_2 - r_1}{2}} \quad , \tag{5a}$$

and in the left semi-region of II,

$$\nabla\phi(r) = \frac{\phi_2 - \phi_b}{\frac{r_3 - r_2}{2}} \quad . \tag{5b}$$

According to assumption 2(b) and Eqs. (5a) and (5b),

$$2\, D_1 \frac{(\phi_b - \phi_1)}{r_2 - r_1} = 2\, D_2 \frac{(\phi_2 - \phi_b)}{r_3 - r_2} \quad , \tag{6}$$

where D_i is the diffusion coefficient in the i^{th} region.

If (6) is solved for the flux at the boundary,

$$\phi_b = \frac{\frac{D_2}{r_3 - r_2}\, \phi_2 + \frac{D_1}{r_2 - r_1}\, \phi_1}{\frac{D_1}{r_2 - r_1} + \frac{D_2}{r_3 - r_2}} \quad . \tag{7}$$

To simplify these relationships, let

$$D_i' = \frac{D_i}{r_{i+1} - r_i} \tag{8}$$

so that

$$\phi_b = \frac{D_2'\, \phi_2 + D_1'\, \phi_1}{D_1' + D_2'} \quad . \tag{9}$$

It is now possible to express $D_1 \nabla\phi$ as a function of ϕ_1 and ϕ_2 by use of Eqs. (5a) and (9).

$$D_1 \nabla \phi = 2\, D_1' \left(\frac{D_2'\, \phi_2 + D_1'\, \phi_1}{D_1' + D_2'} - \phi_1 \right) = \frac{2\, D_1'\, D_2'}{D_1' + D_2'} (\phi_2 - \phi_1) \quad . \tag{10}$$

Equation (10) is valid in any interior region for half of the region. The total diffusion L across the interregional boundaries (for an interior region) is

$$L = \iint D\nabla_n \phi \, dS \cong \left(\frac{2\,D'_{n-1}\,D'_n}{D'_{n-1} + D'_n}\right)(\phi_n - \phi_{n-1})\,S_n + \left(\frac{2\,D'_n\,D'_{n+1}}{D'_n + D'_{n+1}}\right)(\phi_n - \phi_{n+1})\,S_{n+1} \quad , \tag{11}$$

where S_n is the total surface bounding the left side of region n and S_{n+1} is the total surface bounding the right side of region n.

At an outer boundary (see Fig. 3), the inward neutron current is

$$J_- = \frac{\phi_b}{4} + \frac{\lambda_t}{6}\frac{d\phi}{dr} = 0 \quad . \tag{12}$$

The following substitutions can be made in Eq. (12):

(1) $3\,D_n = \lambda_t$;

(2) Multiply λ_t by 1.0656 (correction from transport theory);

(3) $\dfrac{d\phi}{dr} \cong - \dfrac{\phi_n - \phi_b}{(r_{n+1} - r_n)/2}$.

If n = 6, these assumptions and Eq. (12) yield

$$\phi_b = \frac{4.2624\,D'_6}{1 + 4.2624\,D'_6}\,\phi_6 \quad . \tag{13}$$

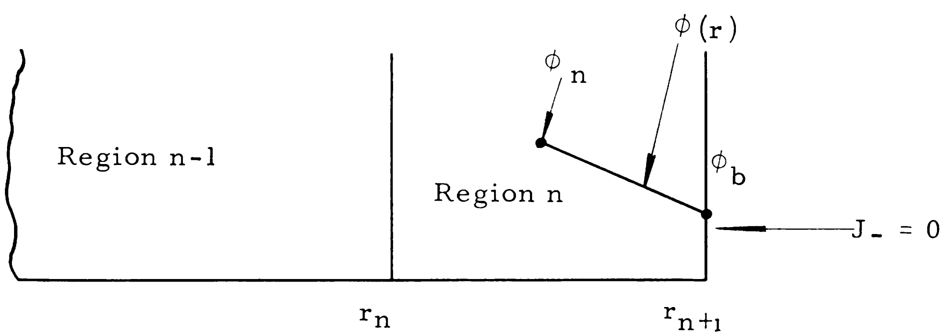


Fig. 3. Flux Variation at the Physical Boundary of the Reactor

The neutron current out of the reactor is

$$J_+ = \frac{\phi_b}{4} - \frac{\lambda_t}{6}\frac{d\phi_b}{dr} \quad . \tag{14}$$

From previous results, if n = 6,

$$J_+ = \frac{2.1312 D_6'}{(1 + 4.2624 D_6')}\,\phi_6 \quad . \tag{15}$$

The total radial leakage out of the reactor is

$$L \cong J_+ S_7 = J\,\phi_6\,S_7 \quad , \tag{16}$$

where

$$J = \frac{2.1312\ D_6'}{1 + 4.2624\ D_6'} \quad . \tag{17}$$

The left boundary of region 1 (see Fig. 4) is a special case. In the left semiregion a zero flux gradient is assumed, so that diffusion must be only considered across S_2 (see Eq. (19) and (23).

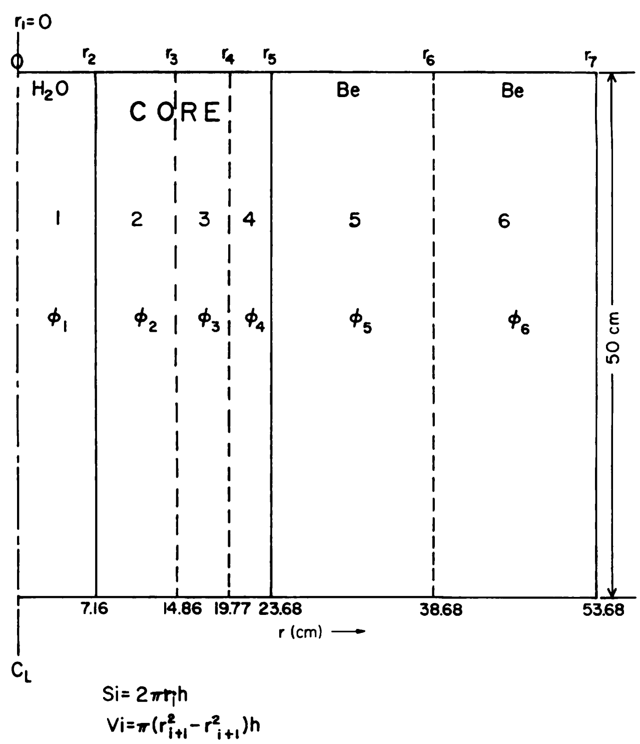


Fig. 4. Reactor Cross Section Showing Spatial Division Used

Now that it is possible to calculate $D\nabla\phi$ in any region for the geometry in question, equations can be written that will yield ϕ as a function of time. Two energy groups are used so that a subscript s represents slow and a subscript f represents fast. The division between the two energy groups is at 0.625 ev.

The reactor shown in Fig. 1 was subdivided into regions as shown in Fig. 4. Two flux equations were solved in each of the regions. In each core region (2, 3, 4), equations were solved for power, fuel depletion, fission products concentration, iodine concentration, xenon concentration, total absorption cross section and control poison cross section.

The analog computer block diagram is shown in Fig. 5.

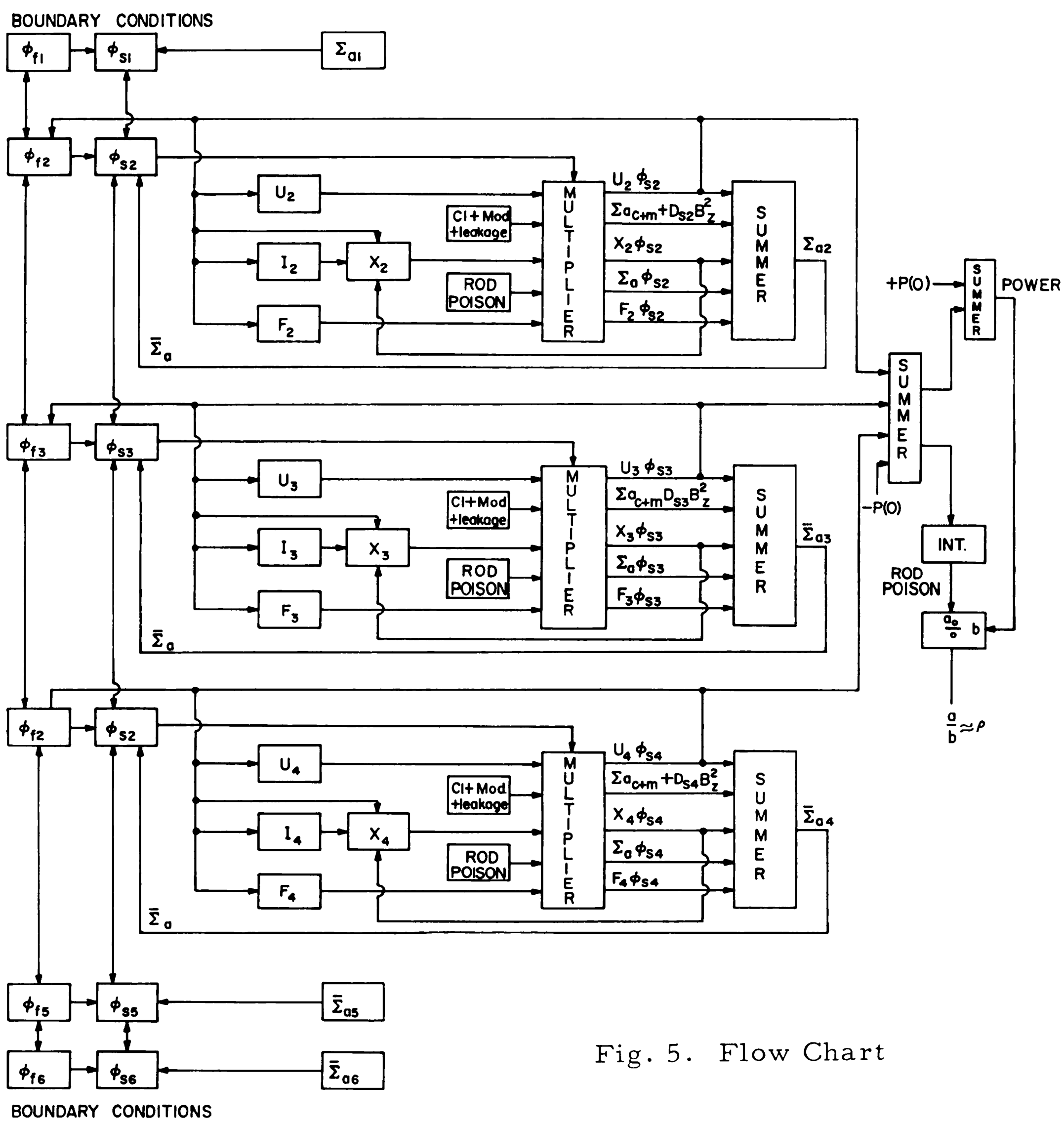


Fig. 5. Flow Chart

For this analysis, the core was divided into three shells of equal volume and the reflector into two shells of equal thickness.

III. FLUX EQUATIONS

Define

$$\mathcal{D}_i = \frac{2D_i' \, D_{i+1}'}{D_i' + D_{i+1}'} \quad . \tag{18}$$

The equations that describe the fast flux are

$$\frac{1}{v_f}\frac{d\phi_{f1}}{dt} V_1 = \left\{ -\mathcal{D}_{f1} S_2 \left(\phi_{f1} - \phi_{f2}\right) - \left(\frac{D_{f1}}{\tau_1} + D_{f1} B_z^2\right) V_1 \phi_{f1} \right\} \quad ; \tag{19}$$

$$\frac{1}{v_f}\frac{d\phi_{f2}}{dt} V_2 = \left\{ -\mathcal{D}_{f1} S_2 \left(\phi_{f2} - \phi_{f1}\right) - \mathcal{D}_{f2} S_3 \left(\phi_{f2} - \phi_{f3}\right) - \left(\frac{D_{f2}}{\tau_2} + D_{f2} B_z^2\right) V_2 \phi_{f2} + \nu \sigma_f U_{235} V_2 \phi_{s2} \right\} \quad ; \tag{20}$$

(The differential equations for ϕ_{f3} and ϕ_{f4} are similar to Eq. (20):

$$\frac{1}{v_f}\frac{d\phi_{f5}}{dt} V_5 = \left\{ -\mathcal{D}_{f4} S_5 \left(\phi_{f5} - \phi_{f4}\right) - \mathcal{D}_{f5} S_6 \left(\phi_{f5} - \phi_{f6}\right) - \left(\frac{D_{f5}}{\tau_5} + D_{f5} B_z^2\right) V_5 \phi_{f5} \right\} \quad ; \tag{21}$$

$$\frac{1}{v_f}\frac{d\phi_{f6}}{dt} V_6 = \left\{ -\mathcal{D}_{f5} S_6 \left(\phi_{f6} - \phi_{f5}\right) - J_{f6} S_7 \phi_{f6} - \left(\frac{D_{f6}}{\tau_6} + D_{f6} B_z^2\right) V_6 \phi_{f6} \right\} \quad . \tag{22}$$

The equations for the slow flux are

$$\frac{1}{v_s}\frac{d\phi_{s1}}{dt} V_1 = \left\{ -\mathcal{D}_{s1} S_2 \left(\phi_{s1} - \phi_{s2}\right) - \left(\Sigma_{a1} + D_{s1} B_z^2\right) V_1 \phi_{s1} + \frac{D_{f1}}{\tau_1} V_1 \phi_{f1} \right\} \quad ; \tag{23}$$

$$\frac{1}{v_s}\frac{d\phi_{s2}}{dt}V_2 = \Big\{-\mathscr{D}_{s1}S_2\ (\phi_{s2} - \phi_{s1}) - \mathscr{D}_{s2}S_3\ (\phi_{s2} - \phi_{s3})$$

$$-\overline{\Sigma}_{a2}\, V_2\ \phi_{s2} + \frac{D_{f2}}{\tau_2}V_2\ \phi_{f2}\Big\} \quad ; \qquad (24)$$

(The differential equations for ϕ_{s3} and ϕ_{s4} are similar to Eq. (24):

$$\frac{1}{v_s}\frac{d\phi_{s5}}{dt}V_5 = \Big\{-\mathscr{D}_{s4}S_5\ (\phi_{s5} - \phi_{s4}) - \mathscr{D}_{s5}S_6\ (\phi_{s5} - \phi_{s6})$$

$$-\left(\Sigma_{a5} + D_{s5}B_z^2\right)\ V_s\ \phi_{s5} + \frac{D_{f5}}{\tau_5}V_5\,\phi_{f5}\Big\} \quad ; \qquad (25)$$

$$\frac{1}{v_s}\frac{d\phi_{s6}}{dt}V_6 = \Big\{-\mathscr{D}_{s5}S_6\ (\phi_{s6} - \phi_{s5}) - J_{s6}S_7\ \phi_{s6}$$

$$-\left(\Sigma_{a6} + D_{s6}B_z^2\right)\ V_6\,\phi_{s6} + \frac{D_{f6}}{\tau_6}V_6\ \phi_{f6}\Big\} \quad . \qquad (26)$$

In the equation for ϕ_{s2}, ϕ_{s3} and ϕ_{s4}, a time-varying absorption term, $\overline{\Sigma}_{ai}$, is introduced.

$$\overline{\Sigma}_{ai} = \sigma_{au}\, U_i + \sigma_{af}\, FP_i + \sigma_{ax}\, X_i + D_{si}\, B_z^2 + \Sigma_{ac} \quad . \qquad (27)$$

The quantities U_i, FP_i, X_i and Σ_{ac} are calculated as functions of time and flux.

$$\Sigma_{ac}(t) \propto \int_0^t \left[\sum_{i=2}^{4} (\sigma_f\ U_i\ V_i\ \phi_{si}) - P_0\right] dt \quad . \qquad (28)$$

$\Sigma_{ac}(t)$ represents the amount of control poison necessary to hold the reactor critical at a given power level P_0. That is, the power level, $\sum\sigma_f\ U_i\ V_i\ \phi_{si}$, is compared continuously with the reference power, P_0, and the integral of the resulting error signal is removed from the initial value of Σ_{ac}, thus maintaining the power at the reference level. This is done via the conventional feedback loop. The presence of this loop is indicated by the occurrence of the term Σ_{ac} on the right hand side of Eq. (27).

Table 2

DIMENSIONS

Sub-script	r_i, (cm)	S_i, (cm^2)	V_i, (cm^3)
1	0	0	8,053
2	7.16	2249	26,633
3	14.86	4668	26,708
4	19.77	6211	26,686
5	23.68	7439	146,932
6	38.68	12152	217,618
7	53.68	16864	

Table 3

REACTOR CONSTANTS

Property		H_2O	Core	Beryllium
D_f	(cm)	1.19	1.1	0.7
τ	(cm^2)	30.0	39.06	72.0
D_s	(cm)	0.1377	0.1684	0.3456
Σ_a	(cm^{-1})	0.01591	changes with time	0.00987
$\Sigma_{a_{clad+mod}}$	(cm^{-1})	0.0	0.08426	0.0
B_z^2 (Including Reflector Savings)	(cm^{-2})	0.00226	0.00226	0.00226
ν = 2.47				
$P_0 = 10^8$ watts				

Element	Concentration at t = 0 (a/cm^3)	$\sigma_f(cm^2)$	$\sigma_a(cm^2)$	γ	λ
Uranium	6.407×10^{20}	3.98×10^{-22}	4.72×10^{-22}	0.0	0.0
Iodine	0.0	0.0	1.2×10^{-24}	0.064	$\frac{0.1033}{3600}$
Xenon	0.0	0.0	3.0×10^{-18}	0.003	$\frac{0.0752}{3600}$
Fission Products	0.0	0.0	$2.5 \times 10^{-23} \frac{cm^2}{fission}$	0.0	0.0

IV. ANALOG COMPUTER CONSIDERATIONS

A. Flux Equations

The time constants in the flux equations are much smaller than the time constants in the equations for control, burnup or poison buildup. Therefore, the solution of the flux equations represents a steady-state solution to the slower equations. In practice, however, the flux equations are almost steady state but are modified by some slowly varying coefficients contributed by the other equations.

Consider Eq. (19),

$$\frac{1}{v_f}\frac{d\phi_{f1}}{dt} V_1 = \left\{ -\mathcal{D}_{f1} S_2 \left(\phi_{f1} - \phi_{f2}\right) - \left(\frac{D_{f1}}{\tau_1} + D_{f1} B_z^2\right) V_1 \phi_{f1} \right\} = Y \quad .$$

Then

$$\frac{d\phi_{f1}}{dt} = \frac{v_f Y}{V_1} \quad . \tag{29}$$

Since v_f is a large number, the actual value of v_f cannot be used. Instead a value G is used, where G is as large as possible. This can only affect the solution in the case of a very rapid transient. For the equations that follow, coefficients will be found in Table 4 together with the auxiliary quantities in Table 5.

$$\frac{d\phi_{f1}}{dt} = G\left(-A_1\phi_{f1} + A_2\phi_{f2}\right) \tag{30}$$

$$\frac{d\phi_{f2}}{dt} = G\left(A_3\phi_{f1} - A_4\phi_{f2} + A_5\phi_{f3} + A_6 U_2 \phi_{s2}\right) \tag{31}$$

$$\frac{d\phi_{f3}}{dt} = G\left(A_7\phi_{f2} - A_8\phi_{f3} + A_9\phi_{f4} + A_{10} U_3 \phi_{s3}\right) \tag{32}$$

$$\frac{d\phi_{f4}}{dt} = G\left(A_{11}\phi_{f3} - A_{12}\phi_{f4} + A_{13}\phi_{f5} + A_{14} U_4 \phi_{s4}\right) \tag{33}$$

$$\frac{d\phi_{f5}}{dt} = G\left(A_{15}\phi_{f4} - A_{16}\phi_{f5} + A_{17}\phi_{f6}\right) \tag{34}$$

$$\frac{d\phi_{f6}}{dt} = G\left(+ A_{18}\phi_{f5} - A_{19}\phi_{f6}\right) \tag{35}$$

$$\frac{d\phi_{s1}}{dt} = G(-B_1\phi_{s1} + B_2\phi_{s2} + B_3\phi_{f1}) \quad (36)$$

$$\frac{d\phi_{s2}}{dt} = G(B_4\phi_{s1} - B_5\phi_{s2} + B_6\phi_{s3} + B_7\phi_{f2} - \overline{\Sigma}_{a2}\phi_{s2}) \quad (37)$$

$$\frac{d\phi_{s3}}{dt} = G(B_8\phi_{s2} - B_9\phi_{s3} + B_{10}\phi_{s4} + B_{11}\phi_{f3} - \overline{\Sigma}_{a3}\phi_{s3}) \quad (38)$$

$$\frac{d\phi_{s4}}{dt} = G(B_{12}\phi_{s3} - B_{13}\phi_{s4} + B_{14}\phi_{s5} + B_{15}\phi_{f4} - \overline{\Sigma}_{a4}\phi_{s4}) \quad (39)$$

$$\frac{d\phi_{s5}}{dt} = G(B_{16}\phi_{s4} - B_{17}\phi_{s5} + B_{18}\phi_{s6} + B_{19}\phi_{f5}) \quad (40)$$

$$\frac{d\phi_{s6}}{dt} = G(B_{20}\phi_{s5} - B_{21}\phi_{s6} + B_{22}\phi_{f6}) \quad (41)$$

Table 4

COEFFICIENTS FOR FLUX EQUATIONS

$A_1 = \mathcal{D}_{f1}\frac{S_2}{V_1} + \frac{D_{f1}}{\tau_1} + D_{f1}B_z^2$	$A_{11} = \mathcal{D}_{f3}\frac{S_4}{V_4}$	$B_1 = \mathcal{D}_{s1}\frac{S_2}{V_1} + \Sigma_{a1} + D_{s1}B_z^2$	$B_{12} = \mathcal{D}_{s3}\frac{S_4}{V_4}$
$A_2 = \mathcal{D}_{f1}\frac{S_2}{V_1}$	$A_{12} = \mathcal{D}_{f3}\frac{S_4}{V_4} + \mathcal{D}_{f4}\frac{S_5}{V_4} + \frac{D_{f4}}{\tau_4} + D_{f4}B_z^2$	$B_2 = \mathcal{D}_{s1}\frac{S_2}{V_1}$	$B_{13} = \mathcal{D}_{s3}\frac{S_4}{V_4} + \mathcal{D}_{s4}\frac{S_5}{V_5}$
$A_3 = \mathcal{D}_{f1}\frac{S_2}{V_2}$	$A_{13} = \mathcal{D}_{f4}\frac{S_5}{V_4}$	$B_3 = \frac{D_{f1}}{\tau_1}$	$B_{14} = \mathcal{D}_{s4}\frac{S_5}{V_4}$
$A_4 = \mathcal{D}_{f1}\frac{S_2}{V_2} + \mathcal{D}_{f2}\frac{S_3}{V_2} + \frac{D_{f2}}{\tau_2} + D_{f2}B_z^2$	$A_{14} = \nu\sigma_{fu}$	$B_4 = \mathcal{D}_{s1}\frac{S_2}{V_2}$	$B_{15} = \frac{D_{f4}}{\tau_4}$
$A_5 = \mathcal{D}_{f2}\frac{S_3}{V_2}$	$A_{15} = \mathcal{D}_{f4}\frac{S_5}{V_5}$	$B_5 = \mathcal{D}_{s1}\frac{S_2}{V_2} + \mathcal{D}_{s2}\frac{S_3}{V_2}$	$B_{16} = \mathcal{D}_{s4}\frac{S_5}{V_5}$
$A_6 = \nu\sigma_{fu}$	$A_{16} = \mathcal{D}_{f4}\frac{S_5}{V_5} + \mathcal{D}_{f5}\frac{S_6}{V_5} + \frac{D_{f5}}{\tau_5} + D_{f5}B_z^2$	$B_6 = \mathcal{D}_{s2}\frac{S_3}{V_2}$	$B_{17} = \mathcal{D}_{s4}\frac{S_5}{V_5} + \mathcal{D}_{s5}\frac{S_6}{V_5} + \Sigma_{a5} + D_{s5}B_z^2$
$A_7 = \mathcal{D}_{f2}\frac{S_3}{V_3}$	$A_{17} = \mathcal{D}_{f5}\frac{S_6}{V_5}$	$B_7 = \frac{D_{f2}}{\tau_2}$	$B_{18} = \mathcal{D}_{s5}\frac{S_6}{V_5}$
$A_8 = \mathcal{D}_{f2}\frac{S_3}{V_3} + \mathcal{D}_{f3}\frac{S_4}{V_3} + \frac{D_{f3}}{\tau_3} + D_{f3}B_z^2$	$A_{18} = \mathcal{D}_{f5}\frac{S_6}{V_6}$	$B_8 = \mathcal{D}_{s2}\frac{S_3}{V_3}$	$B_{19} = \frac{D_{f5}}{\tau_5}$
$A_9 = \mathcal{D}_{f3}\frac{S_4}{V_3}$	$A_{19} = \mathcal{D}_{f5}\frac{S_6}{V_6} + J_{f6}\frac{S_7}{V_6} + \frac{D_{f6}}{\tau_6} + D_{f6}B_z^2$	$B_9 = \mathcal{D}_{s2}\frac{S_3}{V_3} + \mathcal{D}_{s3}\frac{S_4}{V_3}$	$B_{20} = \mathcal{D}_{s5}\frac{S_6}{V_6}$
$A_{10} = \nu\sigma_{fu}$		$B_{10} = \mathcal{D}_{s3}\frac{S_4}{V_3}$	$B_{21} = \mathcal{D}_{s5}\frac{S_6}{V_6} + J_{s6}\frac{S_7}{V_6} + \Sigma_{a6} + D_{s6}B_z^2$
		$B_{11} = \frac{D_{f3}}{\tau_3}$	$B_{22} = \frac{D_{f6}}{\tau_6}$

Table 5

CONSTANTS NEEDED FOR TABLE 4

For "Fast" Equations	For "Slow" Equations	For "Fast" Equations	For "Slow" Equations
$D'_{f1} = \frac{D_{f1}}{r_2 - r_1}$	$D'_{s1} = \frac{D_{s1}}{r_2 - r_1}$	$\mathcal{D}_{f3} = \frac{2\ D'_{f3}\ D'_{f4}}{D'_{f3} + D'_{f4}}$	$\mathcal{D}_{s3} = \frac{2\ D'_{s3}\ D'_{s4}}{D'_{s3} + D'_{s4}}$
$D'_{f2} = \frac{D_{f2}}{r_3 - r_2}$	$D'_{s2} = \frac{D_{s2}}{r_3 - r_2}$	$\mathcal{D}_{f4} = \frac{2\ D'_{f4}\ D'_{f5}}{D'_{f4} + D'_{f5}}$	$\mathcal{D}_{s4} = \frac{2\ D'_{s4}\ D'_{s5}}{D'_{s4} + D'_{s6}}$
$D'_{f3} = \frac{D_{f3}}{r_4 - r_3}$	$D'_{s3} = \frac{D_{s3}}{r_4 - r_3}$	$\mathcal{D}_{f5} = \frac{2\ D'_{f5}\ D'_{f6}}{D'_{f5} + D'_{f6}}$	$\mathcal{D}_{s5} = \frac{2\ D'_{s5}\ D'_{s6}}{D'_{s5} + D'_{s6}}$
$D'_{f4} = \frac{D_{f4}}{r_5 - r_4}$	$D'_{f4} = \frac{D_{s4}}{r_5 - r_4}$	$J_{f6} = \frac{2.1312\ D'_{f6}}{1 + 4.2624\ D'_{f6}}$	$J_{s6} = \frac{2.1312\ D'_{s6}}{1 + 4.2624\ D'_{s6}}$
$D'_{f5} = \frac{D_{f5}}{r_6 - r_5}$	$D'_{f5} = \frac{D_{s5}}{r_6 - r_5}$	$C_2 = \frac{20_{a4}}{a_2 a_3} \sigma_{fu}\ V_2$	
$D'_{f6} = \frac{D_{f6}}{r_7 - r_6}$	$D'_{s6} = \frac{D_{s6}}{r_7 - r_6}$	$C_3 = \frac{20_{a4}}{a_2 a_3} \sigma_{fu}\ V_3$	
$\mathcal{D}_{f1} = \frac{2\ D'_{f1}\ D'_{f2}}{D'_{f1} + D'_{f2}}$	$\mathcal{D}_{s1} = \frac{2\ D'_{s1}\ D'_{s2}}{D'_{s1} + D'_{s2}}$	$C_4 = \frac{20_{a4}}{a_2 a_3} \sigma_{fu}\ V_4$	
$\mathcal{D}_{f2} = \frac{2\ D'_{f2}\ D'_{f3}}{D'_{f2} + D'_{f3}}$	$\mathcal{D}_{s2} = \frac{2\ D'_{s2}\ D'_{s3}}{D'_{s2} + D'_{s3}}$		

B. Fuel and Poison Equations (i = 2,3,4)

$$\frac{dU_i}{dt} = -\sigma_{au}\ U_i \phi_{si} \quad , \quad U_i(0) \neq 0 \tag{42}$$

$$\frac{dFP_i}{dt} = \sigma_{fu}\ U_i \phi_{si} \quad , \quad FP_i(0) = 0 \tag{43}$$

$$\frac{dI_i}{dt} = \gamma_I \sigma_{fu}\ U\phi_{si} - \lambda_I I_i \quad , \quad I_i(0) = 0 \tag{44}$$

$$\frac{dXe_i}{dt} = \gamma_x \sigma_{fu}\ U_i \phi_{si} + \lambda_I I_i - \lambda_x Xe_i - \sigma_{ax}\ Xe_i \phi_{si} \quad , \quad X_i(0) = 0 \tag{45}$$

C. Absorption Equation (i = 2,3,4)

$$\overline{\Sigma}_{ai} = \sigma_{au} U_i + \sigma_{ax} Xe_i + \sigma_{aFP} FP_i + \Sigma_{ac} + \Sigma_{a(c+m)} + D_{si} B_z^2 \qquad (46)$$

D. Power Equation

$$P_0 = 10^8 \text{ megawatts} \qquad (47)$$

E. Control Poison Simulator

$$\Sigma_{ac} = Z_1 \int \left(- \sum_{i=2}^{4} \sigma_{fu} V_i U_i \phi_{si} + P_0 \right) dt + Z_2 \left(- \sum_{i=2}^{4} \sigma_{fu} V_i U_i \phi_{si} + P_0 \right) , \qquad (48)$$

where Z_1 is approximately 100 and Z_2 is unity. Z_2 serves as an initial source and $Z_2 = 0$ after $t > \epsilon$.

F. Definition of Machine Variables

$$t' = a_1 t$$

$$U' = a_2 U$$

$$FP' = a_2 FP$$

$$\phi' = a_3 \phi$$

$$P' = a_4 P$$

$$I' = a_5 I$$

$$Xe' = a_5 Xe$$

G. Values for Scale Factors

$a_1 = \frac{1}{10,800}$	: 1 machine second = 3 hours
$a_2 = 10^{-19}$	: $U = .6407 \times 10^{21}$ a/cm^3 ⟶ 64.07 volts
$a_3 = 10^{-14}$	: $\phi = 10^{16}$ ⟶ 100 volts
$a_4 = 10^{-17}$	: $P_0 = 3.1 \times 10^{18}$ fissions/sec ⟶ 31 volts
$a_5 = 10^{-15}$	: $I = 10^{17}$ a/cm^3 ⟶ 100 volts
	: $Xe = 10^{16}$ a/cm^3 ⟶ 10 volts

H. Scaled Equations (i = 2,3,4)

$$\frac{d\phi'_{f1}}{dt'} = G_1(-A_1\phi'_{f1} + A_2\phi'_{f2}) \quad (30')$$

$$\frac{d\phi'_{f2}}{dt'} = G_1\left(A_3\phi'_{f1} - A_4\phi'_{f2} + A_5\phi'_{f3} + \frac{A_6}{a_2}U'_2\phi'_{s2}\right) \quad (31')$$

$$\frac{d\phi'_{f3}}{dt'} = G_1\left(A_7\phi'_{f2} - A_8\phi'_{f3} + A_9\phi'_{f4} + \frac{A_{10}}{a_2}U'_3\phi'_{s3}\right) \quad (32')$$

$$\frac{d\phi'_{f4}}{dt'} = G_1\left(A_{11}\phi'_{f3} - A_{12}\phi'_{f4} + A_{13}\phi'_{f5} + \frac{A_{14}}{a_2}U'_4\phi'_{s3}\right) \quad (33')$$

$$\frac{d\phi'_{f5}}{dt'} = G_1(A_{15}\phi'_{f4} - A_{16}\phi'_{f5} + A_{17}\phi'_{f6}) \quad (34')$$

$$\frac{d\phi'_{f6}}{dt'} = G_1(A_{18}\phi'_{f5} - A_{19}\phi'_{f6}) \quad (35')$$

$$\frac{d\phi'_{s1}}{dt'} = G_1(-B\phi'_{s1} + B_2\phi'_{s2} + B_3\phi'_{f1}) \quad (36')$$

$$\frac{d\phi'_{s2}}{dt'} = G_1(B_4\phi'_{s1} - B_5\phi'_{s2} + B_6\phi'_{s3} + B_7\phi'_{f2} - \overline{\Sigma}_{a2}\phi'_{s2}) \quad (37')$$

$$\frac{d\phi'_{s3}}{dt'} = G_1(B_8\phi'_{s2} - B_9\phi'_{s3} + B_{10}\phi'_{s4} + B_{11}\phi'_{f3} - \overline{\Sigma}_{a3}\phi'_{s3}) \quad (38')$$

$$\frac{d\phi'_{s4}}{dt'} = G_1(B_{12}\phi'_{s3} - B_{13}\phi'_{s4} + B_{14}\phi'_{s5} + B_{15}\phi'_{f4} - \overline{\Sigma}_{a4}\phi'_{s4}) \quad (39')$$

$$\frac{d\phi'_{s5}}{dt'} = G_1(B_{16}\phi'_{s4} - B_{17}\phi'_{s5} + B_{18}\phi'_{s6} + B_{19}\phi'_{f5}) \quad (40')$$

$$\frac{d\phi'_{s6}}{dt'} = G_1(B_{20}\phi'_{s5} - B_{21}\phi'_{s6} + B_{22}\phi'_{f6}) \quad (41')$$

$$G_1 = \frac{G}{a_1} \quad (49)$$

$$\frac{dU'_i}{dt'} = -\frac{\sigma_{au}}{a_1a_3}U'_i\,\phi'_{si} \quad (42')$$

$$\frac{dFP'_i}{dt'} = \frac{\sigma_{fu}}{a_1a_3}U'_i\phi'_{si} \quad (43')$$

$$\frac{dI_i'}{dt'} = \frac{a_5 \gamma_I \sigma_{fu}}{a_1 a_2 a_3} U_i' \phi_{si}' - \frac{\lambda_I}{a_1} I_i' \tag{44'}$$

$$\frac{dXe_i'}{dt'} = \frac{a_5 \gamma_x \sigma_{fu}}{a_1 a_2 a_3} U_i' \phi_{si}' + \frac{\lambda_I}{a_1} I_i' - \frac{\lambda_x}{a_1} Xe_i' - \frac{\sigma_{ax}}{a_1 a_3} Xe_i' \phi_{si}' \tag{45'}$$

$$\overline{\Sigma}_{ai} \phi_{si}' = \frac{\sigma_{au}}{a_2} U_i' \phi_{si}' + \frac{\sigma_{ax}}{a_5} Xe_i' \phi_{si}' + \frac{\sigma_{aFP}}{a_2} FP_i' \phi_{si} + [D_{si} B_z^2 + \Sigma_{a(c+m)}] \phi_{si} + \Sigma_{ac} \phi_{si}' \tag{46'}$$

Note: i = 2,3,4

$$P_0' - \sum_{i=2}^{4} \frac{a_4 \sigma_{fu} V_i}{a_2 a_3} U_i' \phi_{si}' = 0 \tag{47'}$$

$$\Sigma_{ac} = Z_1 \int - (P'(t) - P_0')\, dt' + Z_2 (-P'(t) + P_0') \tag{48'}$$

I. Machine Equations (i = 2,3,4)

$$\frac{d\phi_{f1}'}{dt'} = G_2(-.8527\, \phi_{f1}' + .4291\, \phi_{f2}') \tag{30''}$$

$$\frac{d\phi_{f2}'}{dt'} = G_2\left(.1298\, \phi_{f1}' - .742\, \phi_{f2}' + .3058\, \phi_{f3}' + 1.966 \frac{\phi_{s2}' U_2'}{20}\right) \tag{31''}$$

$$\frac{d\phi_{f3}'}{dt'} = G_2\left(.3049\, \phi_{f2}' - 1.1915\, \phi_{f3}' + .5801\, \phi_{f4}' + 1.966 \frac{\phi_{s3}' U_3'}{20}\right) \tag{32''}$$

$$\frac{d\phi_{f4}'}{dt'} = G_2\left(.5805\, \phi_{f3}' - 1.1102\, \phi_{f4}' + .2232\, \phi_{f5}' + 1.966 \frac{\phi_{s4}' U_4'}{20}\right) \tag{33''}$$

$$\frac{d\phi_{f5}'}{dt'} = G_2(.04053\, \phi_{f4}' - .1922\, \phi_{f5}' + .0386\, \phi_{f6}') \tag{34''}$$

$$\frac{d\phi_{f6}'}{dt'} = G_2(.02606\, \phi_{f5}' - .2033\, \phi_{f6}') \tag{35''}$$

$$\frac{d\phi_{s1}'}{dt'} = G_2(-.2194\, \phi_{s1}' + .05716\, \phi_{s2}' + .3967\, \phi_{f1}') \tag{36''}$$

$$\frac{d\phi_{s2}'}{dt'} = G_2(.01728\, \phi_{s1}' - .0641\, \phi_{s2}' + .04681\, \phi_{s3}' + .2816\, \phi_{f2}' - 10\overline{\Sigma}_{a2} \phi_{s2}' \tag{37''}$$

$$\frac{d\phi'_{s3}}{dt'} = G_2(.04668\,\phi'_{s2} - .1355\,\phi'_{s3} + .0888\,\phi'_{s4} + .2816\,\phi'_{f3} - 10\overline{\Sigma}_{a3}\phi'_{s3}) \tag{38''}$$

$$\frac{d\phi'_{s4}}{dt'} = G_2(.0889\,\phi'_{s3} - .1041\,\phi'_{s4} + .0837\,\phi'_{s5} + .2816\,\phi'_{f4} - 10\overline{\Sigma}_{a4}\phi'_{s3}) \tag{39''}$$

$$\frac{d\phi'_{s5}}{dt'} = G_2(.0152\,\phi'_{s4} - .1408\,\phi'_{s5} + .0191\,\phi'_{s6} + .0972\,\phi'_{f5}) \tag{40''}$$

$$\frac{d\phi'_{s6}}{dt'} = G_2(.0129\,\phi'_{s5} - .1540\,\phi'_{s6} + .0972\,\phi'_{f6}) \tag{41''}$$

$$G_2 = \frac{G_1}{10} \tag{49'}$$

$$\frac{d\,U'_i}{dt'} = -.0102\left(\frac{U'_i \cdot \phi'_{s2}}{20}\right) \tag{42''}$$

$$\frac{dFP'_i}{dt'} = 0.0086\left(\frac{FP'_i \cdot \phi'_{si}}{20}\right) \tag{43''}$$

$$\frac{dI'_i}{dt'} = 5.502\left(\frac{U'_i \cdot \phi'_{si}}{20}\right) - .3099\,I'_i \tag{44''}$$

$$\frac{dXe'_i}{dt'} = .2579\left(\frac{U'_i \cdot \phi'_{si}}{20}\right) + .3099\,I'_i - .2256\,Xe'_i - 6.48\left(\frac{Xe'_i\phi'_{si}}{2}\right) \tag{45''}$$

$$10\overline{\Sigma}_{ai}\phi'_{si} = 0.944\left(\frac{U' \cdot \phi'_{si}}{20}\right) + .0600\left(\frac{Xe'_i\phi'_{si}}{2}\right) + 0.050\left(\frac{FP' \cdot \phi'_{si}}{20}\right) + .8652\,\phi_{si} + \left(\frac{200\Sigma_{ac}\phi'_{si}}{20}\right) \tag{46''}$$

$$P'_0 - \sum_{i=2}^{4} C_i\left(\frac{U_i \cdot \phi'_{si}}{20}\right) = 0 \tag{47''}$$

$$\Sigma_{ac} = 100\int (P'(t) - P'_0)\,dt' + (P'(t) - P_0) \tag{48''}$$

J. Circuit Diagram

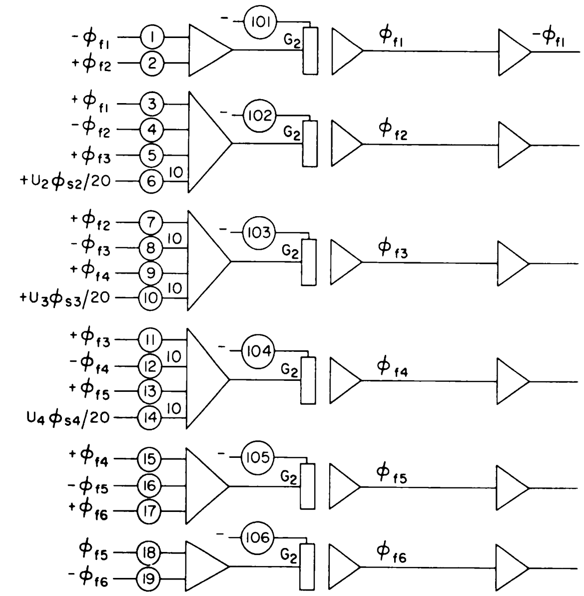


Fig. 6. Simulator for Fast Flux

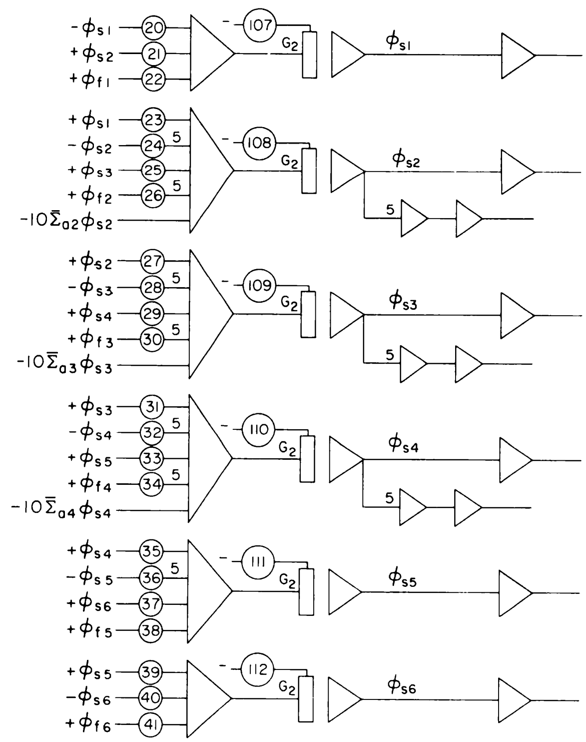


Fig. 7. Simulator for Slow Flux

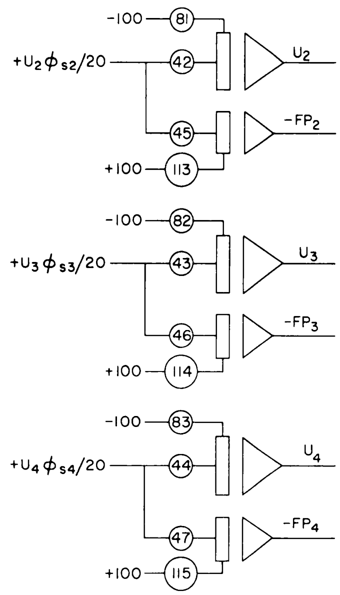


Fig. 8. Fuel Burnout and Fission Product Buildup Simulator

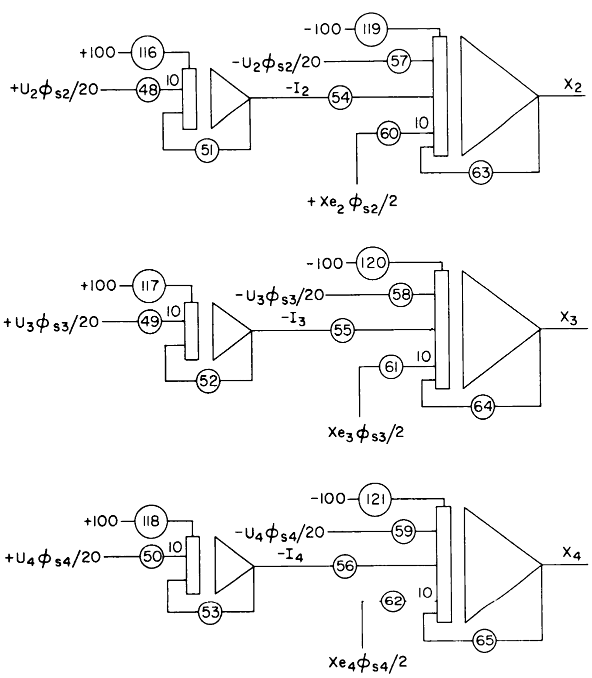


Fig. 9. Iodine-Xenon Simulator

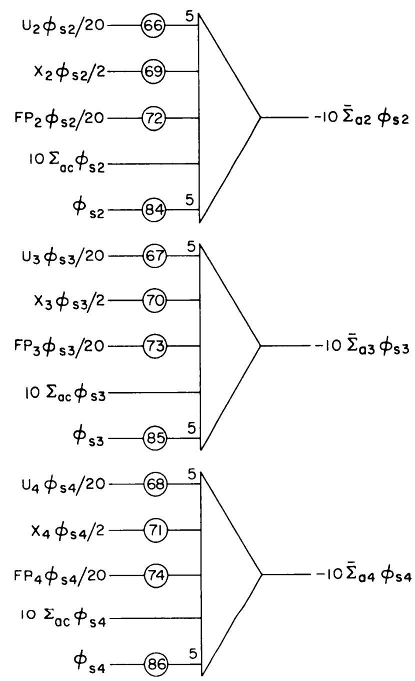


Fig. 10. Absorption Calculator

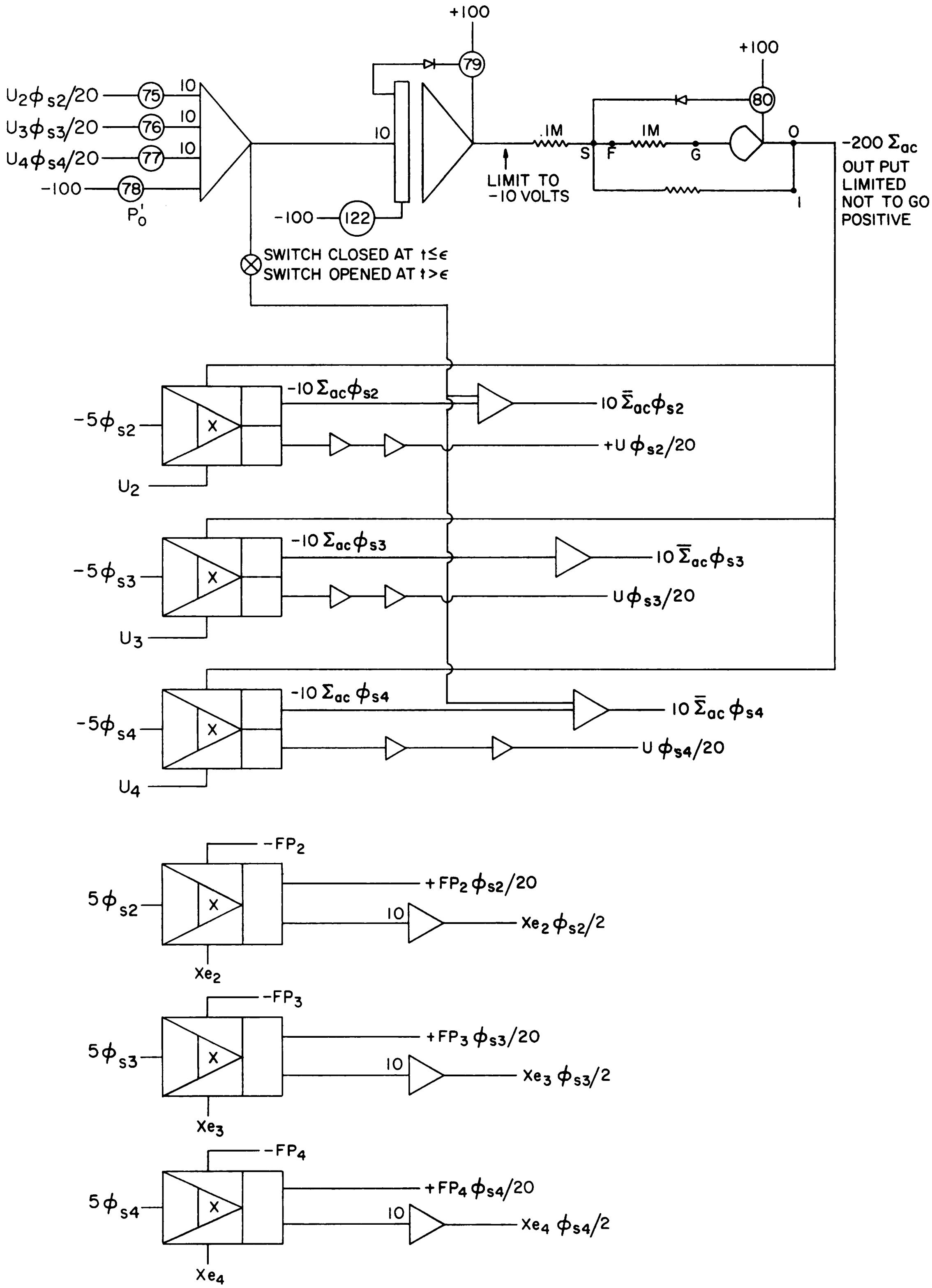


Fig. 11. Control Poison Calculator and Multipliers

K. Potentiometer Settings

Potentiometer No.	Input	Math. Value	Value	Setting	G
1	$-\phi_{f1}$	$10\ A_1$	.8527	8527	
2	ϕ_{f2}	$10\ A_2$	.4291	4291	
3	ϕ_{f1}	$10\ A_3$	.1298	1298	
4	$-\phi_{f2}$	$10\ A_4$	.742	7420	
5	ϕ_{f3}	$10\ A_5$	.3058	3058	
6	$U_2\phi_{s2}/20$	$200\ A_6/a_2$	1.9662	1966	10
7	ϕ_{f2}	$10\ A_6$	.3049	3049	
8	$-\phi_{f3}$	$10\ A_8$	1.1915	1192	10
9	ϕ_{f4}	$10\ A_9$	.5801	5801	
10	$U_3\phi_{s3}/20$	$200\ A_{10}/a_2$	1.966	1966	10
11	ϕ_{f3}	$10\ A_{11}$	.5805	5805	
12	$-\phi_{f4}$	$10\ A_{12}$	1.1102	1110	10
13	ϕ_{f5}	$10\ A_{13}$	.2232	2232	
14	$U_4\phi_{s4}/20$	$200\ A_{14}/a_2$	1.996	1966	10
15	ϕ_{f4}	$10\ A_{15}$	.04053	0405	
16	$-\phi_{f5}$	$10\ A_{16}$	.1922	1922	
17	ϕ_{f6}	$10\ A_{17}$	.0386	0386	
18	ϕ_{f5}	$10\ A_{18}$	.02606	0261	
19	$-\phi_{f6}$	$10\ A_{19}$	.2033	2033	
20	$-\phi_{s1}$	$10\ B_1$	.2194	2194	
21	ϕ_{s2}	$10\ B_2$	.05716	0572	
22	ϕ_{f1}	$10\ B_3$	.3967	3967	
23	ϕ_{s1}	$10\ B_4$	.01728	0173	
24	$-\phi_{s2}$	$10\ B_5$	.0641	0128	5
25	ϕ_{s3}	$10\ B_6$	.04681	0468	
26	ϕ_{f2}	$10\ B_7$	.2816	0563	5
27	ϕ_{s2}	$10\ B_8$	.04668	0467	
28	$-\phi_{s3}$	$10\ B_9$	.1355	0271	5

Potentiometer No.	Input	Math. Value	Value	Setting	G
29	ϕ_{s4}	$10\ B_{10}$	.0888	0888	
30	ϕ_{f3}	$10\ B_{11}$	.2816	0563	5
31	ϕ_{s3}	$10\ B_{12}$	.0889	0889	
32	$-\phi_{s4}$	$10\ B_{13}$	.1041	0208	5
33	ϕ_{s5}	$10\ B_{14}$	.0837	0837	
34	ϕ_{f4}	$10\ B_{15}$	.2816	0563	5
35	ϕ_{s4}	$10\ B_{16}$	.0152	0152	
36	$-\phi_{s5}$	$10\ B_{17}$	.1408	0282	5
37	ϕ_{s6}	$10\ B_{18}$	.0191	0191	
38	ϕ_{f5}	$10\ B_{19}$	.0972	0972	
39	ϕ_{f5}	$10\ B_{20}$	.0129	0129	
40	$-\phi_{s6}$	$10\ B_{21}$	.1540	1540	
41	ϕ_{f6}	$10\ B_{22}$	.0972	0972	
42	$+U_2\phi_{s2}/20$	$20\ \sigma_{au}/a_1a_3$	.0102	0102	
43	$U_3\phi_{s3}/20$		.0102	0102	
44	$U_4\phi_{s4}/20$		.0102	0102	
45	$U_2\phi_{s2}/20$	$20\ \sigma_{fu}/a_1a_3$	.0086	0086	
46	$U_3\phi_{s3}/20$		.0086	0086	
47	$U_4\phi_{s4}/20$		.0086	0086	
48	$U_2\phi_{s2}/20$	$20\ a_5\gamma_I\sigma_{fu}/a_1a_2a_3$	5.502	5502	10
49	$U_3\phi_{s3}/20$		5.502	5502	10
50	$U_4\phi_{s4}/20$		5.502	5502	10
51	$-I_2$	λ_I/a_1	.3099	3099	
52	$-I_3$		.3099	3099	
53	$-I_4$		.3099	3099	
54	$-I_2$		.3099	3099	
55	$-I_3$		.3099	3099	
56	$-I_4$		.3099	3099	
57	$-U_2\phi_{s2}/20$	$20\ a_5\gamma_x\sigma_{fu}/a_1a_2a_3$	.2579	2579	
58	$-U_3\phi_{s3}/20$		.2579	2579	

Potentiometer No.	Input	Math. Value	Value	Setting	G
59	$-U_4\phi_{s4}/20$		.2579	2579	
60	$Xe_2\phi_{s2}/2$	$2\,\sigma_{ax}/a_1a_3$	6.48	6480	10
61	$Xe_3\phi_{s3}/20$		6.48	6480	10
62	$Xe_4\phi_{s4}/20$		6.48	6480	10
63	X_2	λ_x/a_1	.2256	2256	
64	X_3		.2256	2256	
65	X_4		.2256	2256	
66	$U_2\phi_{s2}/20$	$200\,\sigma_{au}/a_2$	.944	1888	5
67	$U_3\phi_{s3}/20$		.944	1888	5
68	$U_4\phi_{s4}/20$		.944	1888	5
69	$X_2\phi_{s2}/2$	$20\,\sigma_{ax}/a_5$	.06	0600	
70	$X_3\phi_{s3}/2$		.06	0600	
71	$X_4\phi_{s4}/2$		.06	0600	
72	$FP_2\phi_{s2}/20$	$200\,\sigma_{aFP}/a_2$	.05	0500	
73	$FP_3\phi_{s3}/20$		.05	0500	
74	$FP_4\phi_{s4}/20$		.05	0500	
75	$U_2\phi_{s2}/20$	C_2	2.12	2120	10
76	$U_3\phi_{s3}/20$	C_3	2.126	2126	10
77	$U_4\phi_{s4}/20$	C_4	2.124	2124	10
78	-100	P_0'	31.00	3100	
79		Limit		9135 dial reading	
80		Limit		9500 dial reading	
81	-100	$U_2'(0)$	64.07	6407	
82	-100	$U_3'(0)$	64.07	6407	
83	-100	$U_4'(0)$	64.07	6407	
84	ϕ_{s2}	$10\,(\Sigma a\,(c+m) + D_{s2}B_z^2)$	.8652	1730	5
85	ϕ_{s3}		.8652	1730	5
86	ϕ_{s4}		.8652	1730	5
101 to 122	used for initial conditions when starting from $t \neq 0$				

V. ANALOG RESULTS

The answers to the test problem discussed in Section IV are given graphically in this section.

Figures 12 and 13 give the fast and slow flux levels at the points indicated by the subscript 1 - 6. Note that the slow flux at point 1 is shown with the fast fluxes in Fig. 12. This comes about because of the much higher level of flux at this point, which is representative of the internal thermal column.

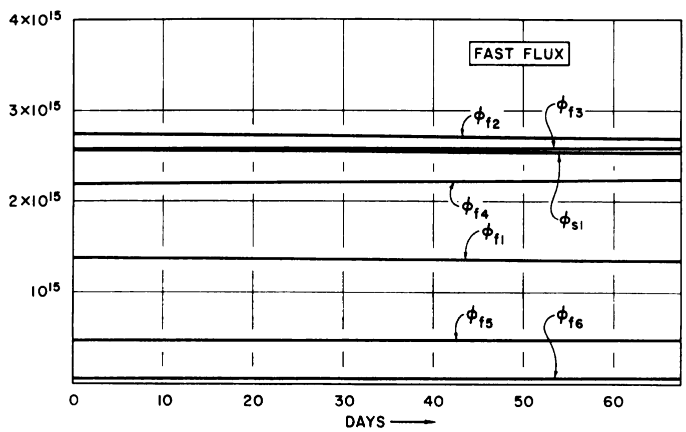


Fig. 12. Fast Flux vs t

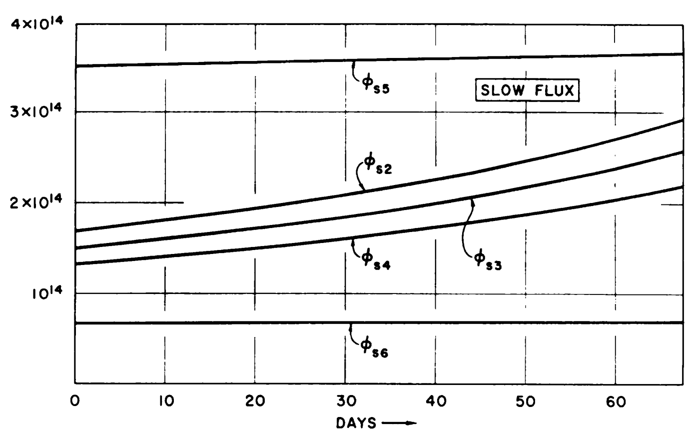


Fig. 13. Slow Flux vs t

Σ_{ac}, the indicated control poison, and ρ, the reactivity held down by Σ_{ac}, are given in Figs. 14 and 15.

Isotope number densities in the core regions are given for U^{235}, fission products, and xenon in Figs. 16-18.

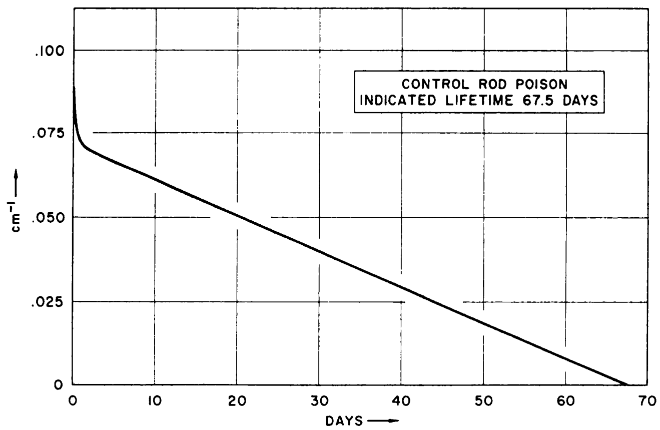


Fig. 14. Σ_{ac} vs t

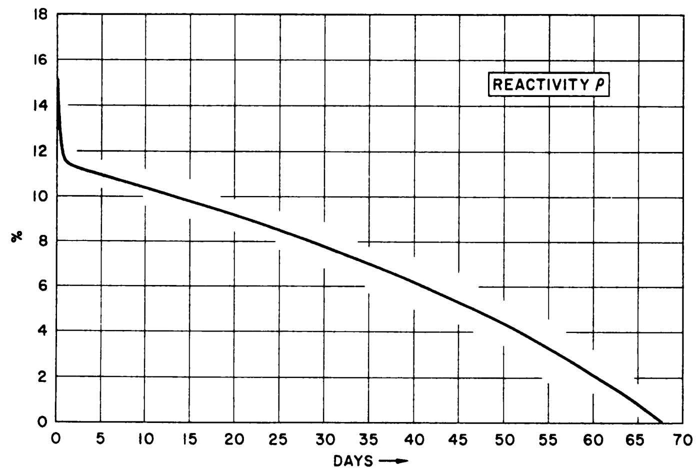


Fig. 15. Reactivity vs t

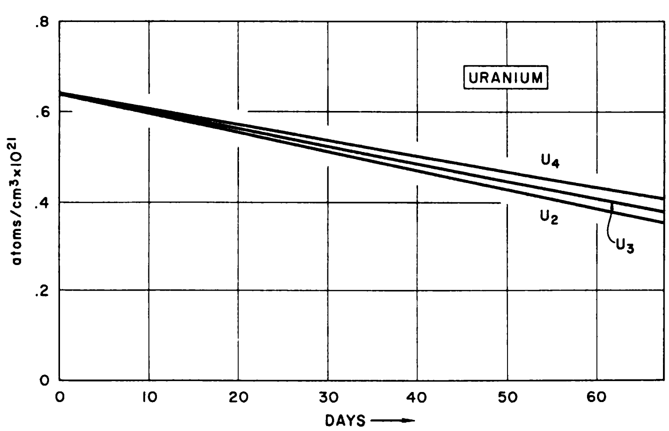


Fig. 16. Uranium Concentration vs t

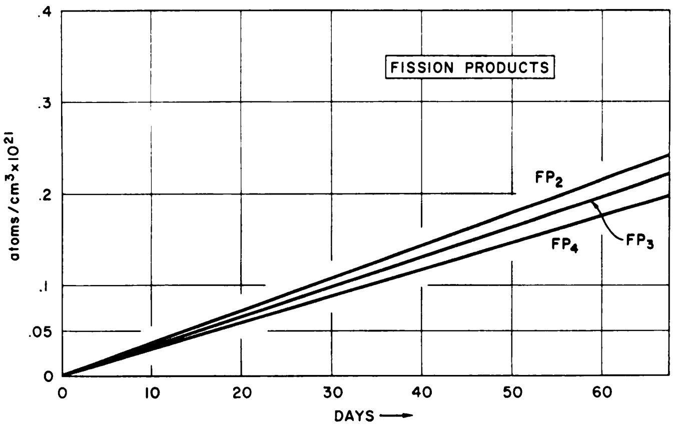


Fig. 17. Fission Product Concentration vs t

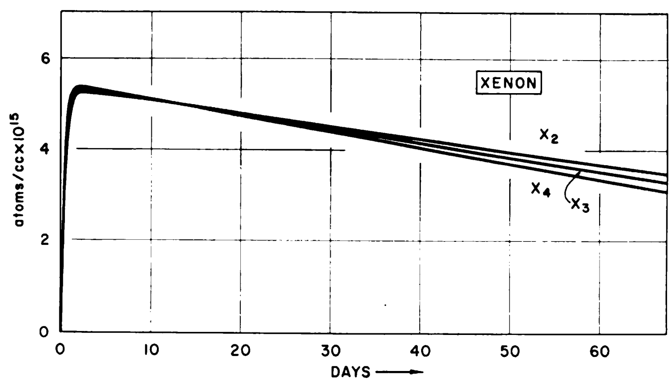


Fig. 18. Xenon Concentration vs t

The control poison (Σ_{ac}) and the xenon number densities in region 3 following a set back in power after 60 hours of steady operation are given in Figs. 19-23. The title "Set-Back to 10%" means the reactor power (P_0) is reduced to 0.1 P_0 (= 10 Mw in the test problem). Figure 24 represents the effect of a complete shutdown (P_0 reduced to 0) at 60 hours.

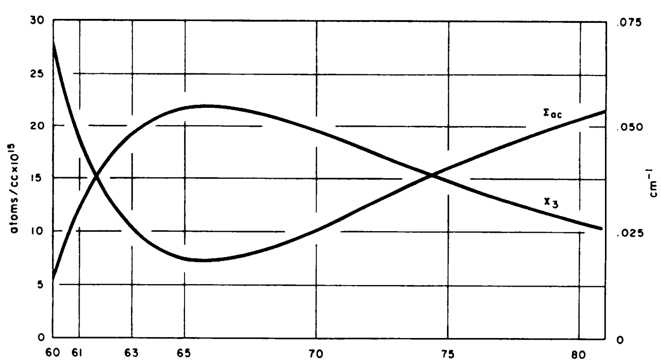


Fig. 19. Xenon-Σ_{ac} Transient Initiated by a Set-Back to 10 Mw

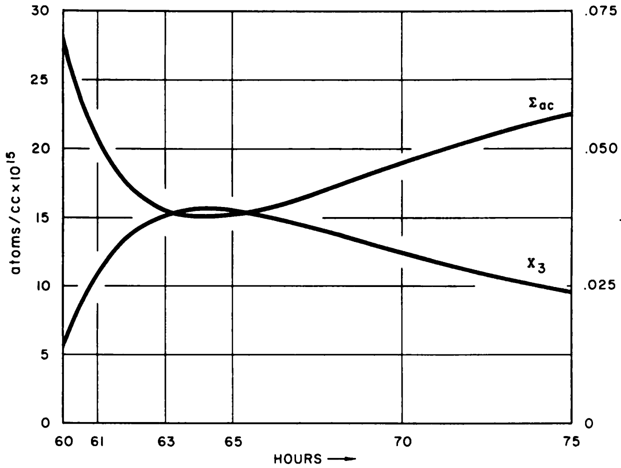
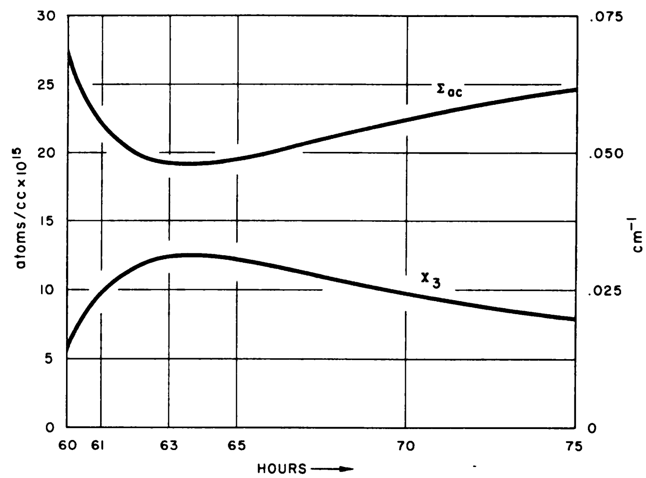


Fig. 20

Xenon-Σ_{ac} Transient Initiated by a Set-Back to 20 Mw

Fig. 21

Xenon-Σ_{ac} Transient Initiated by a Set-Back to 30 Mw



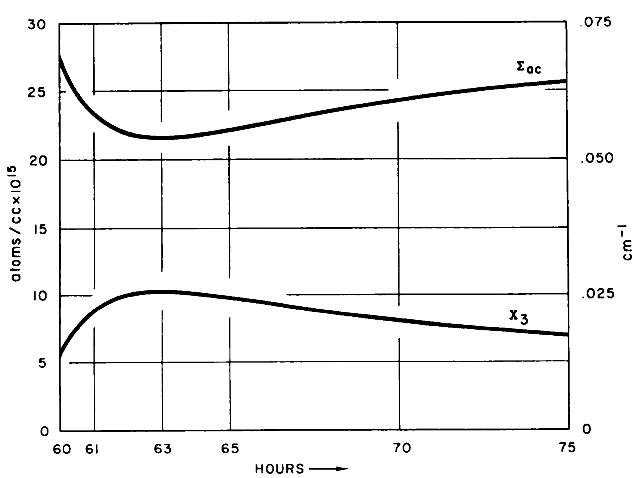


Fig. 22

Xenon-Σ_{ac} Transient Initiated by a Set-Back to 40 Mw

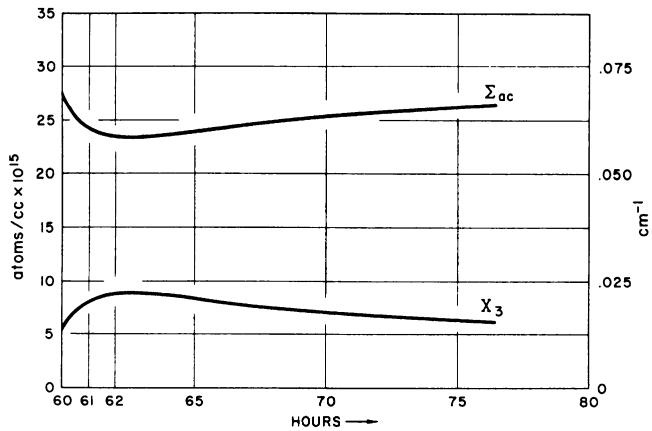


Fig. 23. Xenon-Σ_{ac} Transient Initiated by a Set-Back to 50 Mw

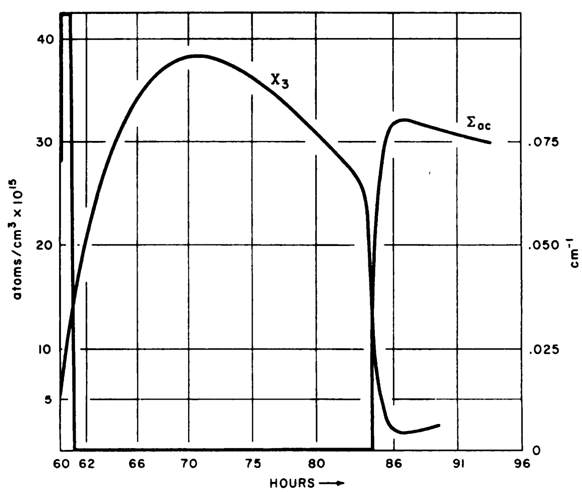


Fig. 24. Xenon- Σ_{ac} Transient Initiated by 100% Set-Back at 60 hrs. Full Power Demanded at 70 hrs.

At 70 hours P_0 was restored to its initial level. The reactor could not go critical until approximately 85 hours. The initial use and sharp decline in Σ_{ac} represents a transient voltage in the problem caused by switching P_0 to zero. It is not a real effect.

VI. COMPARISON OF ANALOG AND DIGITAL RESULTS

The flux levels computed at t = 0 by the analog program are compared with those computed by a standard Argonne digital code, RE 122, in Fig. 25. The flux levels computed after 67.5 days by the analog program are also shown in Fig. 25.

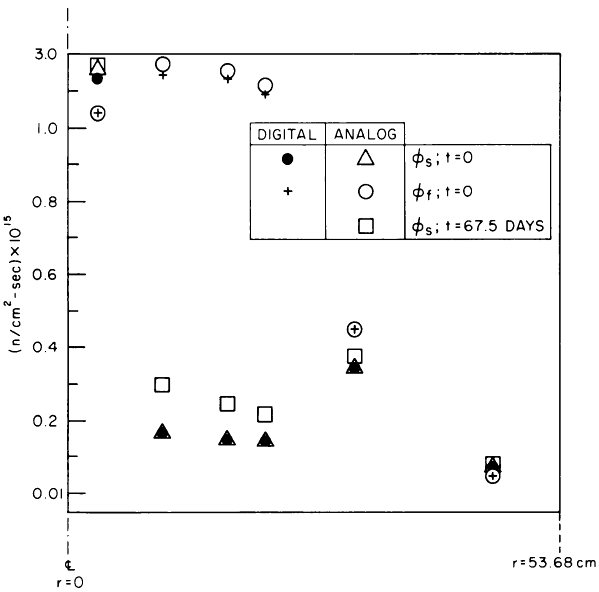


Fig. 25. Flux vs. Distance from Center of Reactor at t = 0 and t = 67.5 days

The agreement of the flux levels calculated by the two methods is good, especially in view of the small number of spatial points used in the analog computer solution and the large flux gradients observed. As would be expected with so crude a description of the flux shape, the reactivity calculated by the analog program does not agree with the result of the digital program.

A detailed examination of the discrepancy reveals that the relatively crude estimate of the leakage in the analog program causes the reactivity decrement from leakage to be over-estimated. Since no burnup occurs in the outer region, the flux gradient here would be better estimated by the insertion of another spatial point near the outer boundary.

The test problem is admittedly an extreme case with large flux gradients. Problems typical of power reactor designs do not usually exhibit such large gradients and the error in the leakage term will then be much smaller.

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